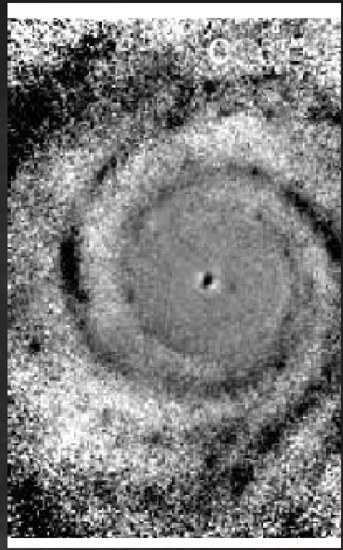
31st SUMMER SCHOOL – CONFERENCE:
DYNAMIC SYSTEMS AND COMPLEXITY

# HAMILTONIAN DYNAMICS AND STRUCTURE FORMATION IN DISK GALAXIES

Panos Patsis
RCAAM, Academy of Athens

# 1991

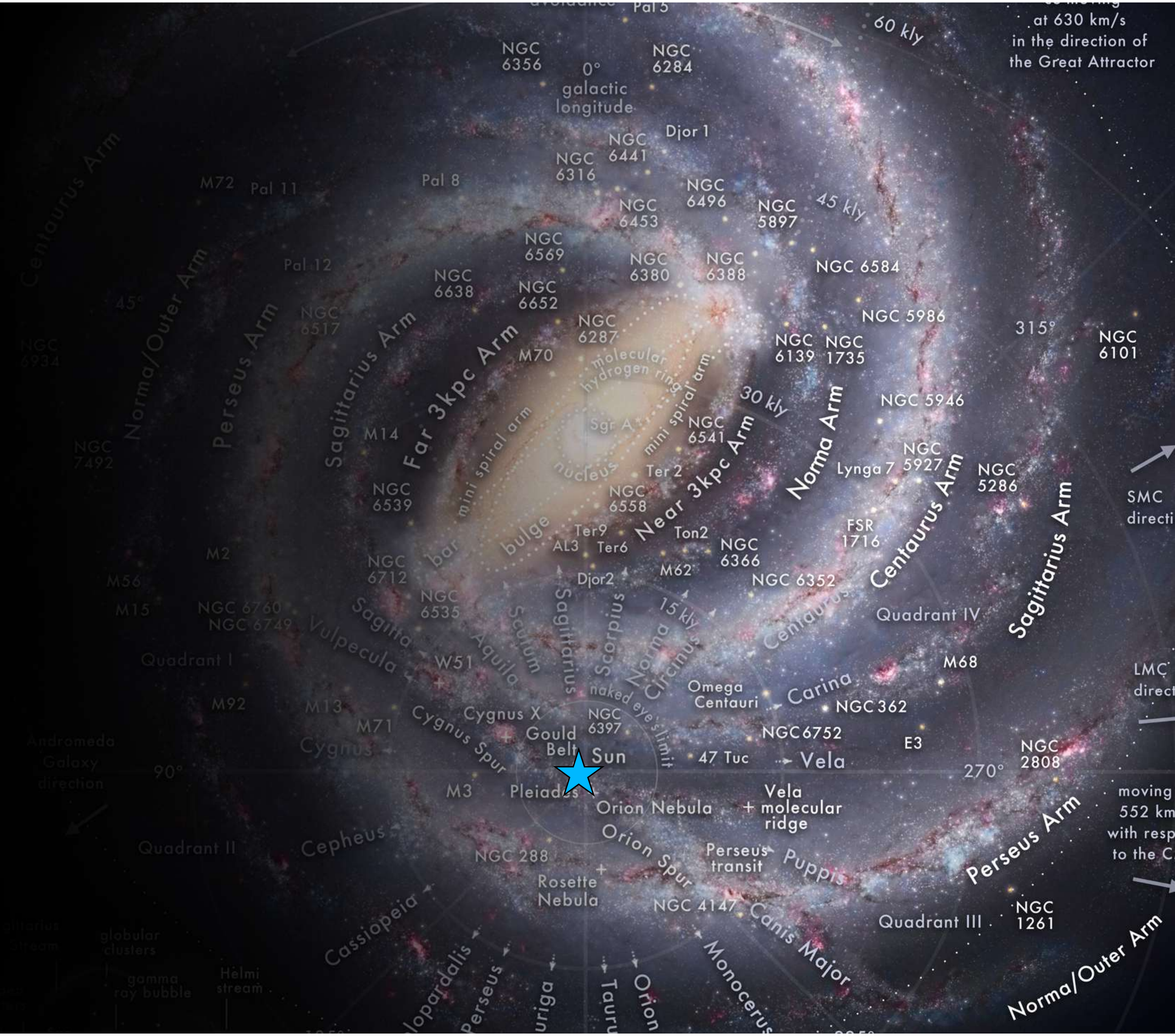
     

# Morphology is the result of stellar and gaseous flows

- NGC2997 (ESO – optical) / Hawk-I (P.Grosbol 2010)
- NGC986 (HST) / Ks (Buta et al. 2009)
- NGC3223(CGS) /K' (Grosbol & Patsis 1998)

## Bars (and structures in general) in galaxies, are not only a riddle per se

- *Distribution of dark and luminous matter*
- *Global star formation (planet formation)*
- *Bars transfer matter from the edges of the galaxy to their center.*
- ***The MW is a barred-spiral galaxy***

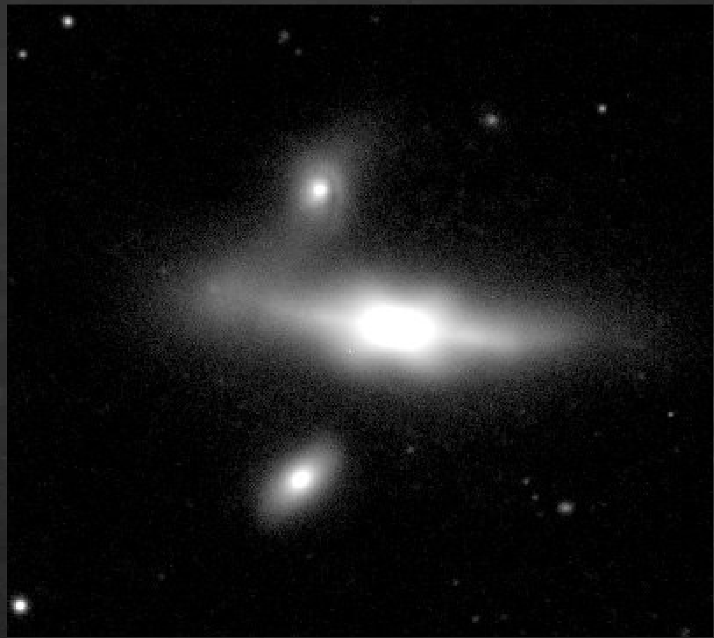


# Bars

- NGC 1300 (VLT, HAWK-I)

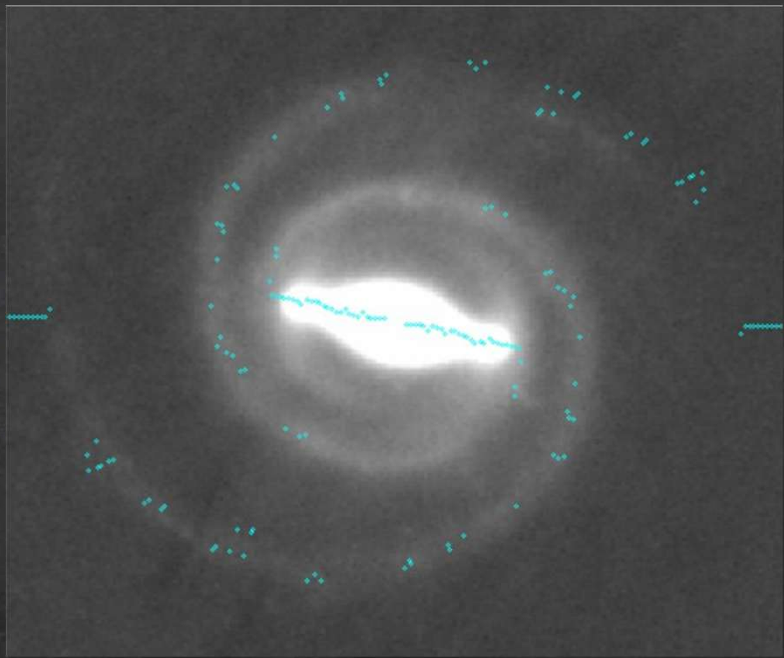
Grosbol 2010

NGC 128 (2.3m Aristarchos)



P.+ Xilouris, Alikakos

# Fourier Analysis to describe the observed structures
Patsis, Alikakos, Xilouris 20205 (in prep.)



# Orbital theory & Nonlinear phenomena

- In the early 1960s, Orbital theory emerged, marking the beginning of modeling galaxies with autonomous Hamiltonians (Contopoulos etc.).
- Over the subsequent decades, significant findings were obtained from this approach, which continue to hold true to this day. At the very least, these findings have laid the groundwork for a more profound comprehension of the dynamics governing barred-spiral galaxies.

# Orbits in rotating barred potentials

Equations of motion are derived from the Hamiltonian

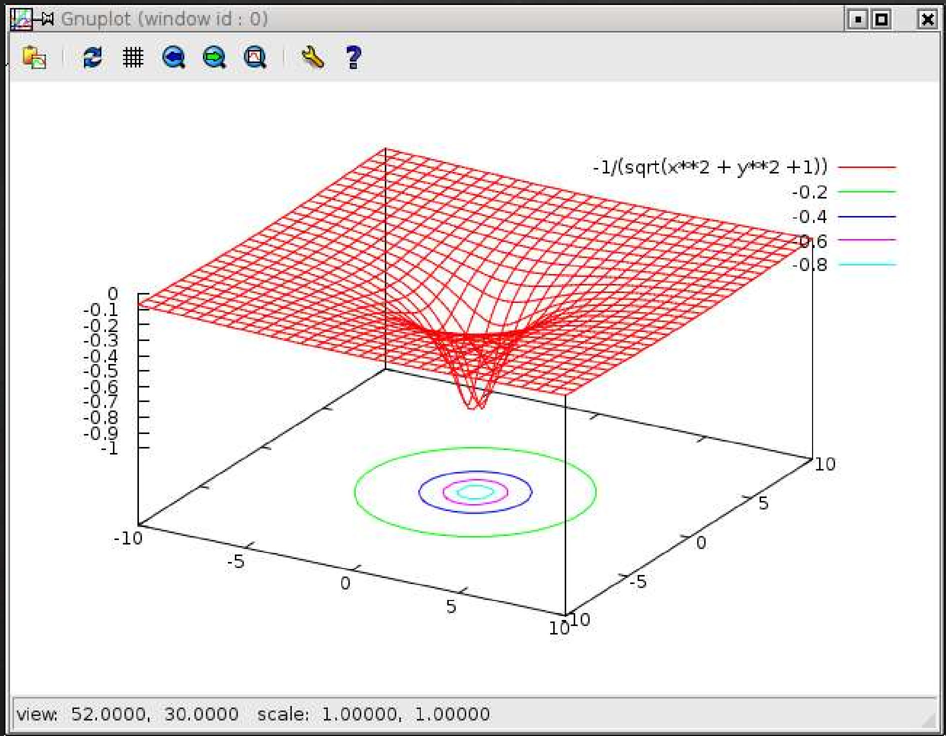
$$H \equiv \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \Phi(x,y) - \frac{1}{2}\Omega_s^2(x^2 + y^2) = E_J$$

where $(x, y)$ are the coordinates in a Cartesian frame of reference corotating with the spiral with angular velocity $\Omega_s$. $\Phi(x, y)$ is the potential in Cartesian coordinates, $E_J$ is the numerical value of the Jacobian integral and dots denote time derivatives.

Effective potential

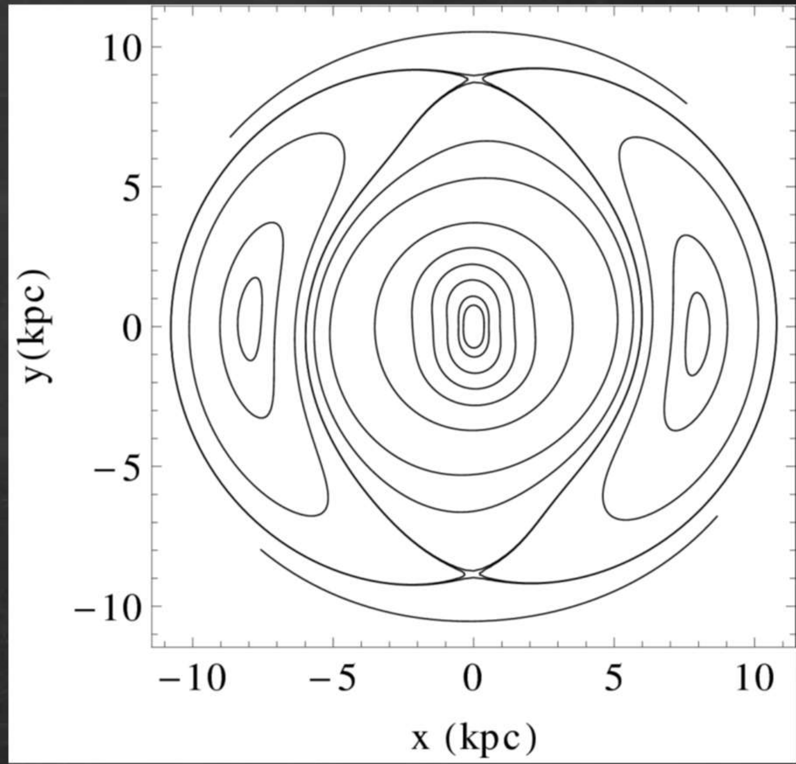
$E_J$, the Jacobi integral, is the rotating-frame analog of the total energy

## Potential



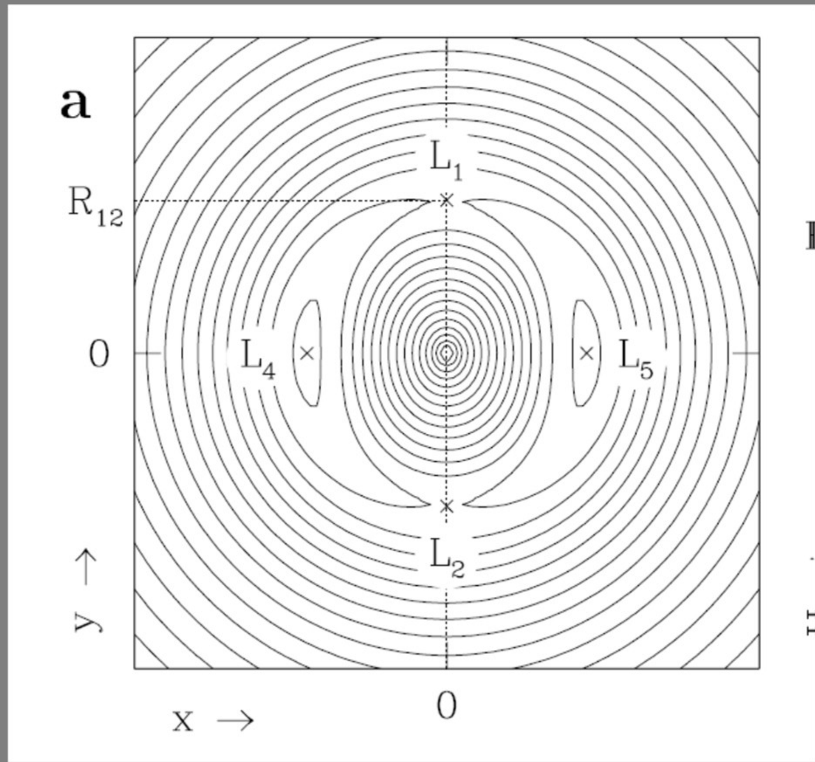

## Effective Potential

## Equilibrium points



# Barred potentials

Equilibrium points




$A(r)$

$$\Phi = \Phi_0 + A\cos(2\theta) - \Omega_p J_o$$

# Potentials: I. Analytic

Ferrers bar:

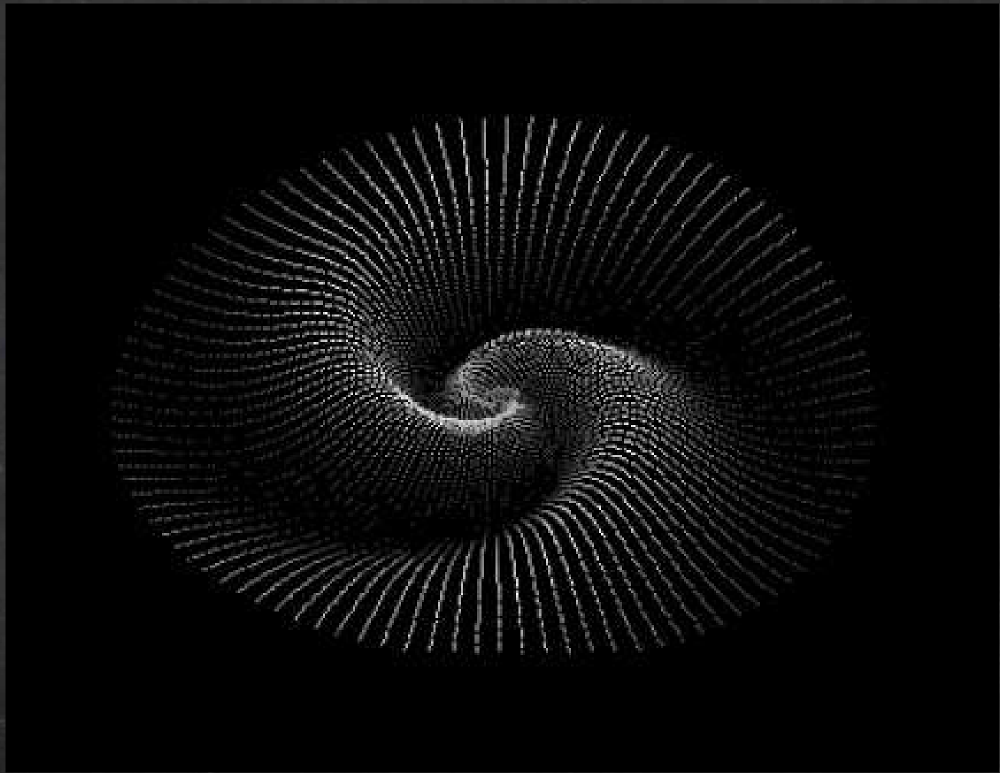
Q. J. Pure Appl. Math. 1877, 14, 1

$$\rho = \begin{cases} \dfrac{105 M_B}{32\pi abc}(1-m^2)^2 & \text{for} \quad m \leqq 1 \\ 0 & \text{for} \quad m > 1 \end{cases},$$

$$m^2 = \frac{y^2}{a^2} + \frac{x^2}{b^2} + \frac{z^2}{c^2}\,, \quad a > b > c,$$

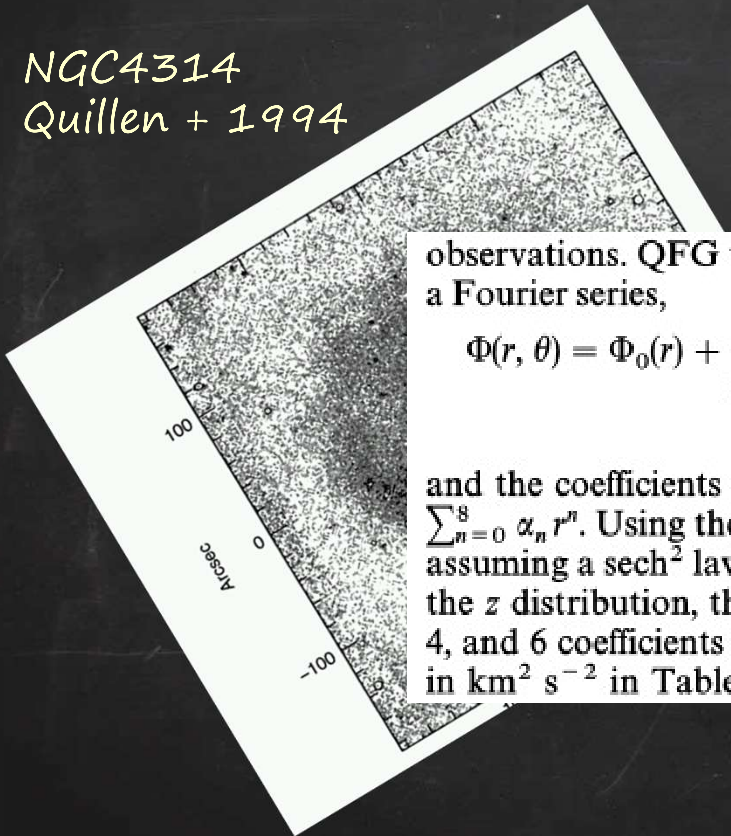
$$V_1(r,\theta) = Ar\exp(-\varepsilon_s r)\cos(2\ln r/\tan i_0 - 2\theta)$$

Contopoulos & Grosbol 1986

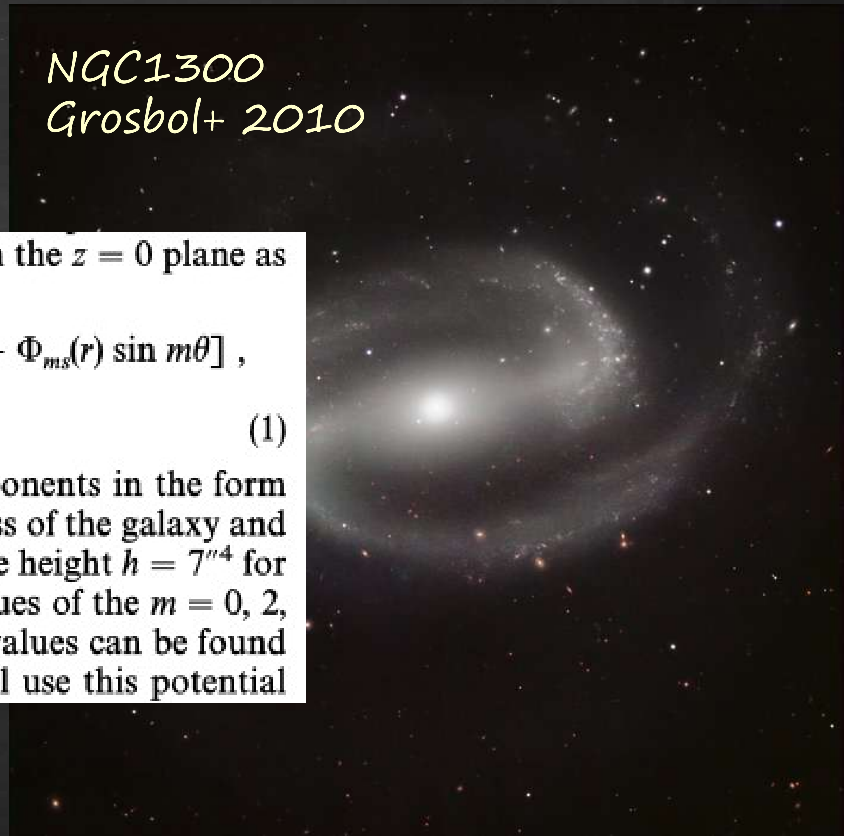


# Potentials: II – Estimated from NIR images of glaxies

NGC4314
Quillen + 1994




NGC1300
Grosbol+ 2010

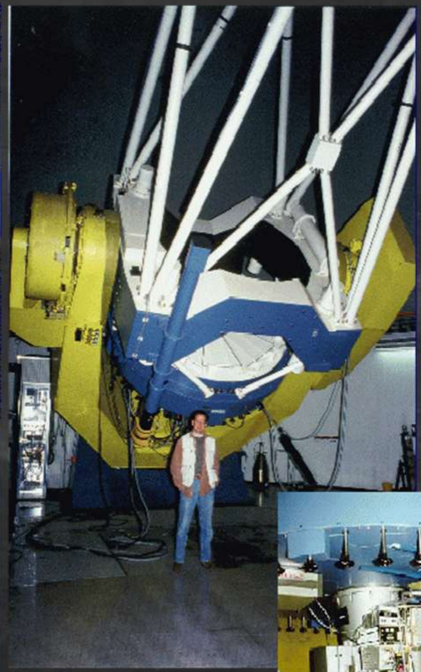


observations. QFG write the potential in the $z = 0$ plane as a Fourier series,

$$\Phi(r, \theta) = \Phi_0(r) + \sum_{m>0} [\Phi_{mc}(r) \cos m\theta + \Phi_{ms}(r) \sin m\theta] , \tag{1}$$

and the coefficients of the various components in the form $\sum_{n=0}^{8} \alpha_n r^n$. Using the $K$ surface brightness of the galaxy and assuming a sech$^2$ law with a vertical scale height $h = 7''4$ for the $z$ distribution, they calculate the values of the $m = 0$, 2, 4, and 6 coefficients $\alpha_n$ (their numerical values can be found in km$^2$ s$^{-2}$ in Table 1 of QFG). We will use this potential

# Near-infrared observations in the '90s

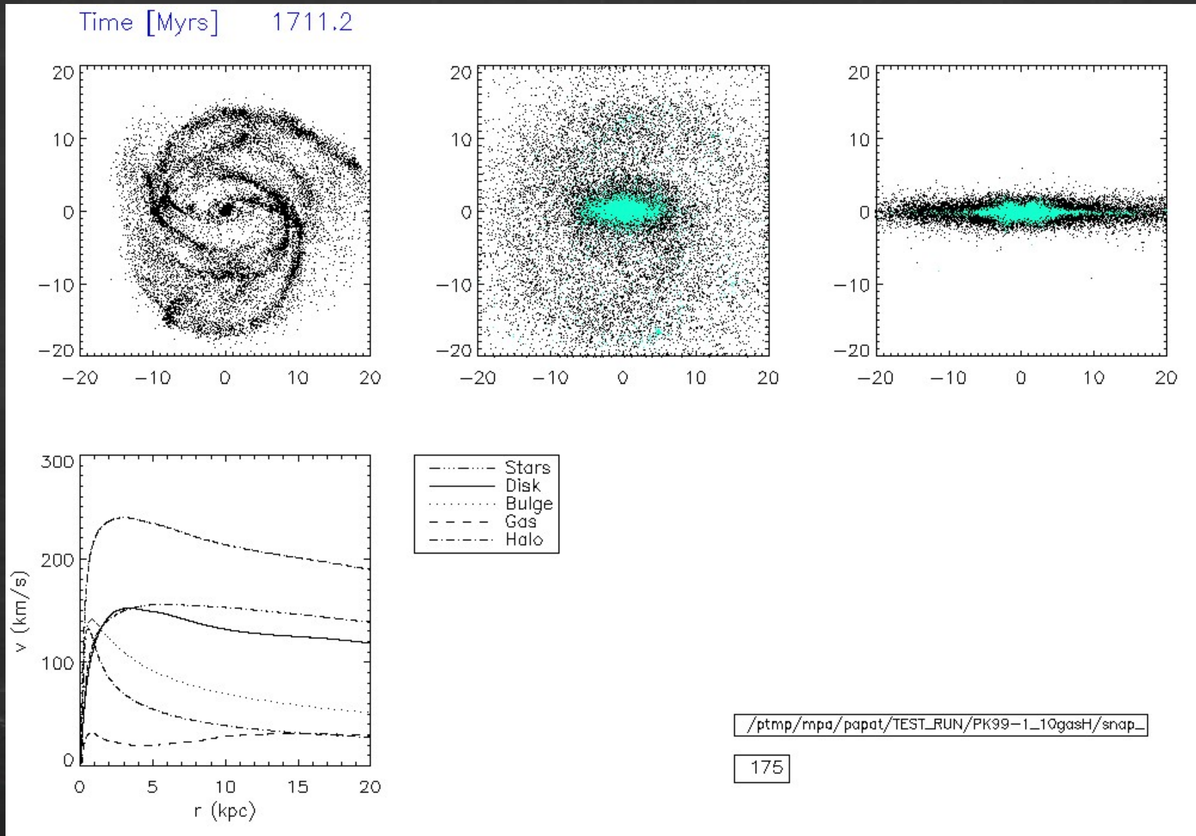



# Are there non-barred, grand-design spiral galaxies?

...just with an "oval distortion"

Could all spirals be chaotic? (due to Lyapunov orbits)

# Potentials: III – From N-body simulations




Patsis & Naab
in preparation

# Resonances (axisymmetric case)

resonances between the epicyclic frequency $\kappa$ and the angular velocity in the rotating frame $(\Omega - \Omega_s)$ (where $\Omega$ and $\Omega_s$ are the angular velocities of the stars and of the spiral pattern), i.e. when

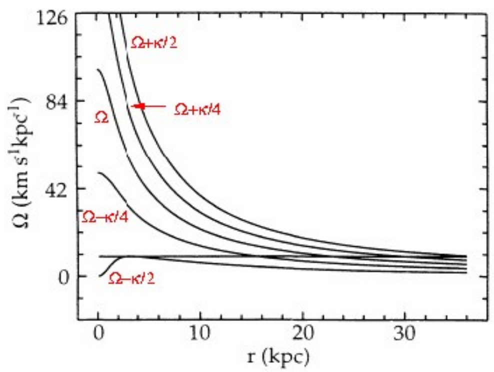
$$\frac{\overset{\nearrow}{\kappa}}{\underset{\text{V}}{\Omega - \Omega_s}} = \frac{n}{m}.$$

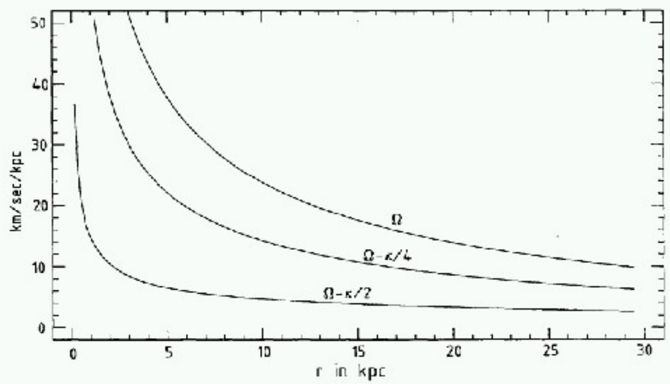
***n/m = 2/1 Inner Lindblad Resonance***
***= 4/1 Inner 4:1 (UHR)***

$\Omega = \Omega_S$ ***COROTATION***

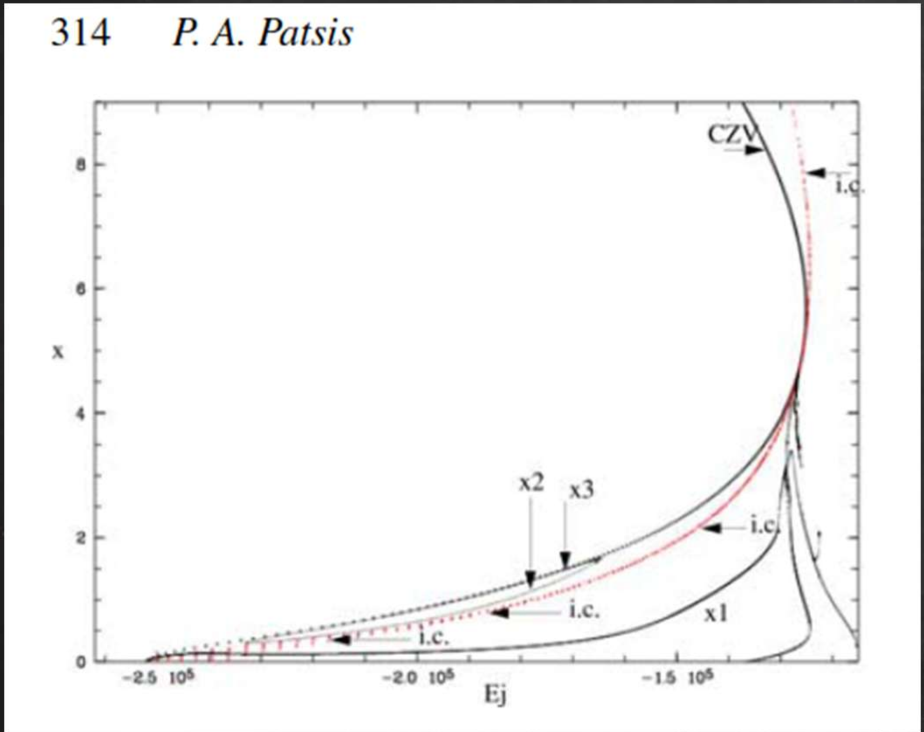
***n/m = –4/1 Outer 4:1***

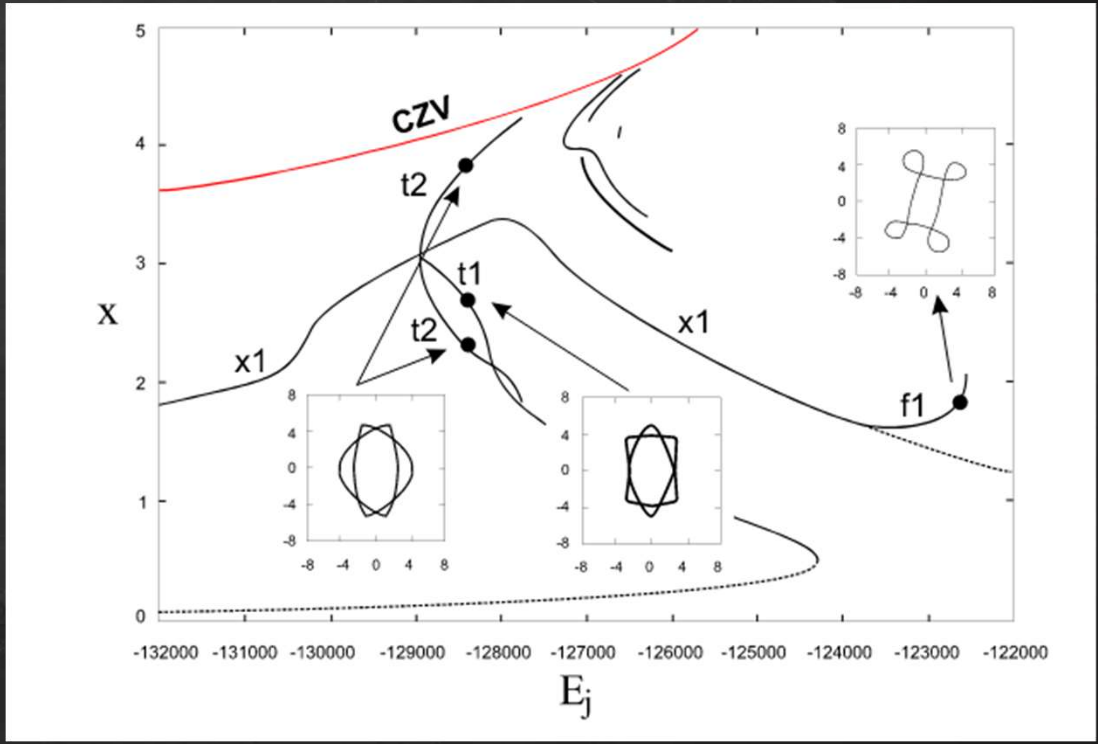
***= –2/1 Outer Lindblad Resonance***



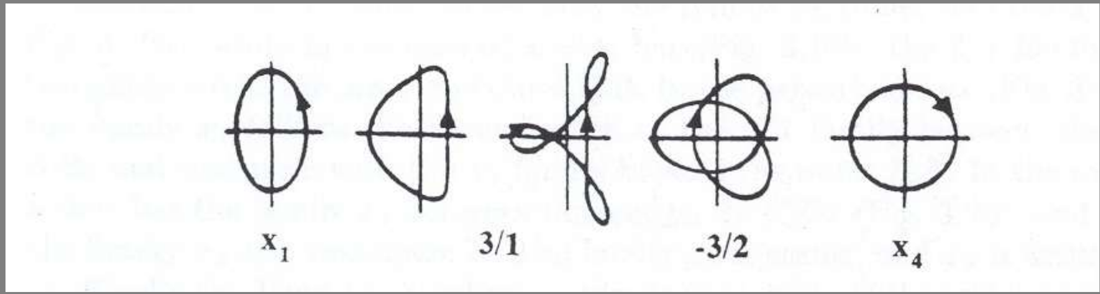
# 1st tool: The characteristic

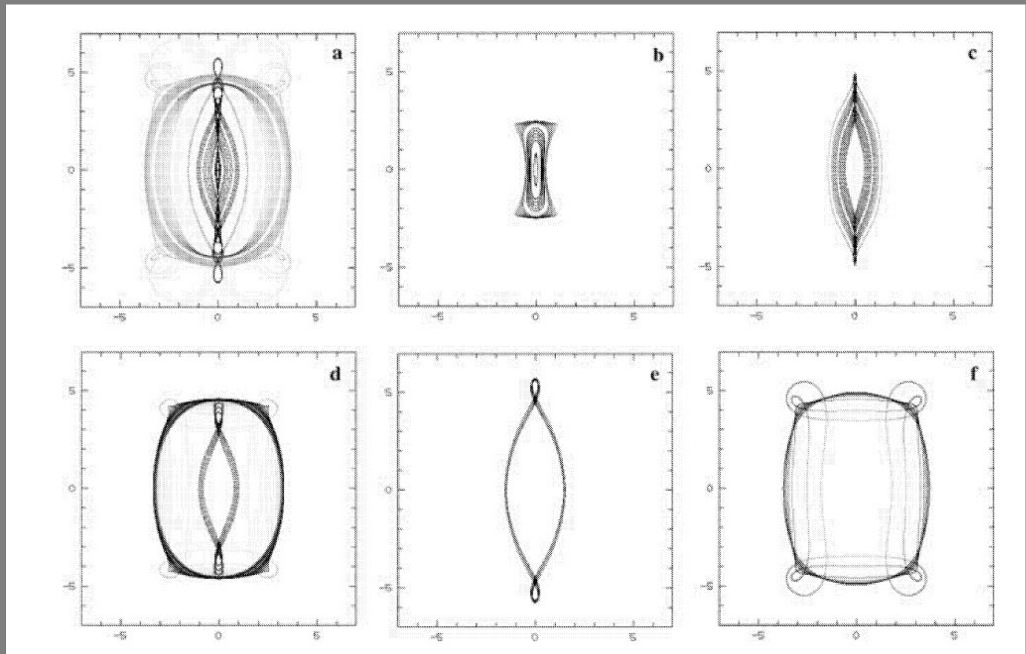


# Morphological evolution of p.o.'s with Ej

Contopoulos+ '60s-'80s
Athanassoula '80s-90's




Skokos,Patsis,
Athanassoula 2002

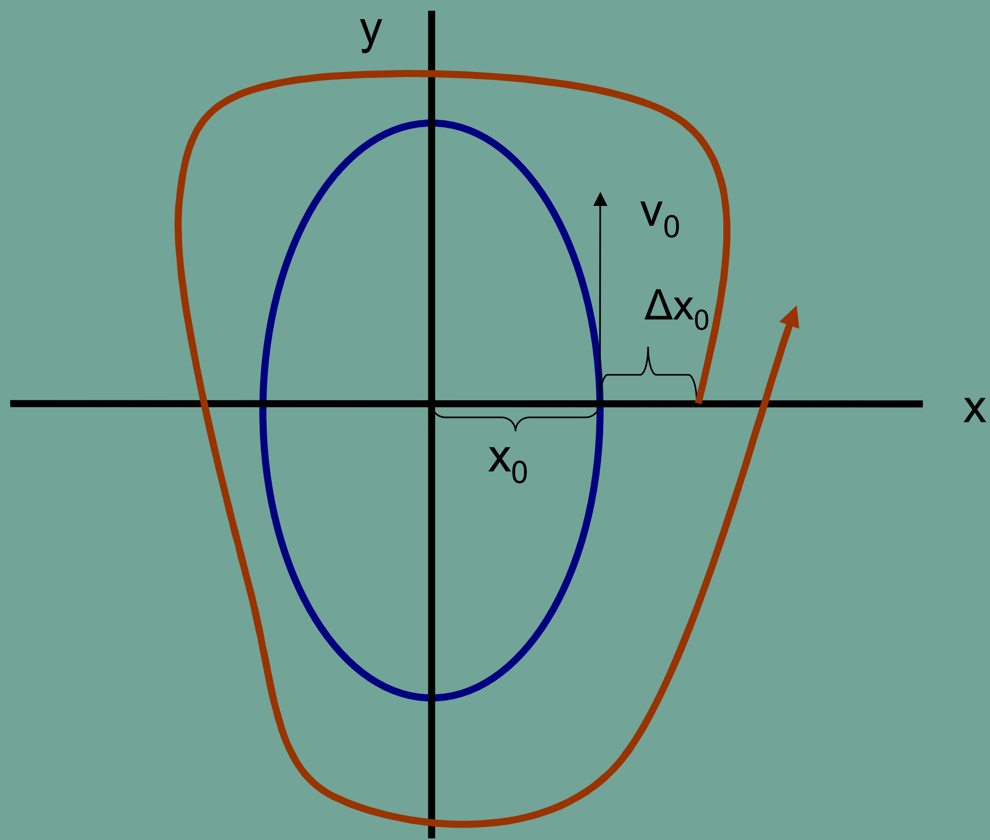
# Non-integrable systems
# Orbits

Orbits: **periodic** – non periodic

**Periodic** orbits. : Stable - Unstable

$$x = x_0(t) \quad + \xi$$

# Stability of periodic orbits (Hénon 1965)



$$\boldsymbol{y = 0}, \qquad \dot{\mathrm{y}} > \mathbf{0}$$

$$\boldsymbol{H = H(x, 0, \dot{x}, \dot{y}) = h \rightarrow y = \cdots}$$

$g = (g_1, g_2)$

$g = (g\colon \mathbb{R}^2 \rightarrow \mathbb{R}^2,\ \vec{\xi} = \mathrm{M}\vec{\xi}_0$ *(relates final with initial displacements)*

M: *monodromy matrix (2x2)*

*Characteristic equation:* $\lambda^2 - (a + d)\lambda + 1 = 0,\ \left(a = \frac{\partial g_1}{\partial x},\ d = \frac{\partial g_2}{\partial \dot{x}}\right)$

*Henon index:* $\alpha = \frac{1}{2}(a + d)$

*p.o. STABLE* $\Leftrightarrow |\alpha| < 1$ ($|\lambda_1| = |\lambda_2|$, *complex*)

*p.o. UNSTABLE* $\Leftrightarrow |\alpha| > 1$ ($\lambda_1 \lambda_2 = 1$, *real*)

# X1 family

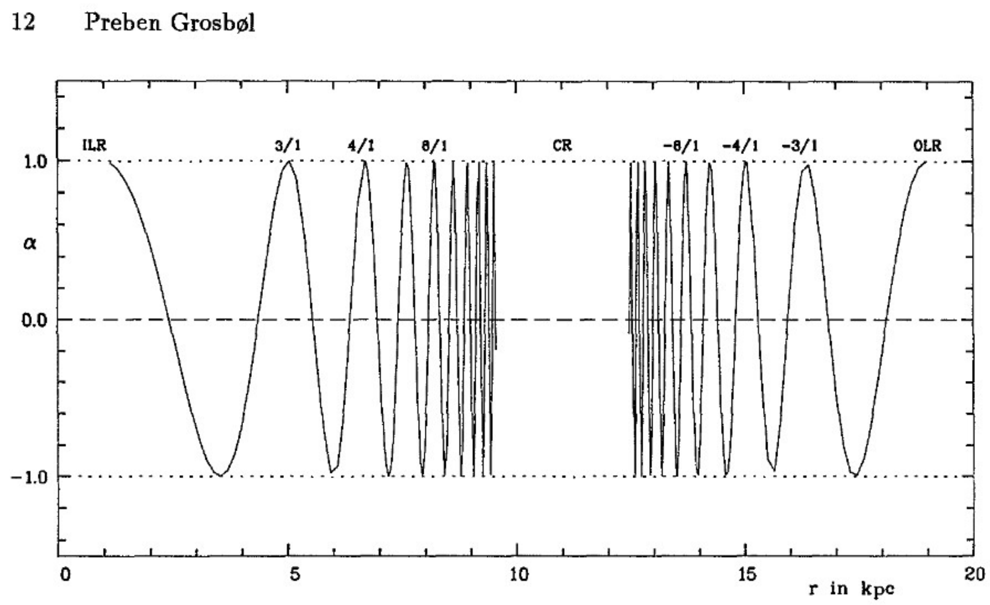
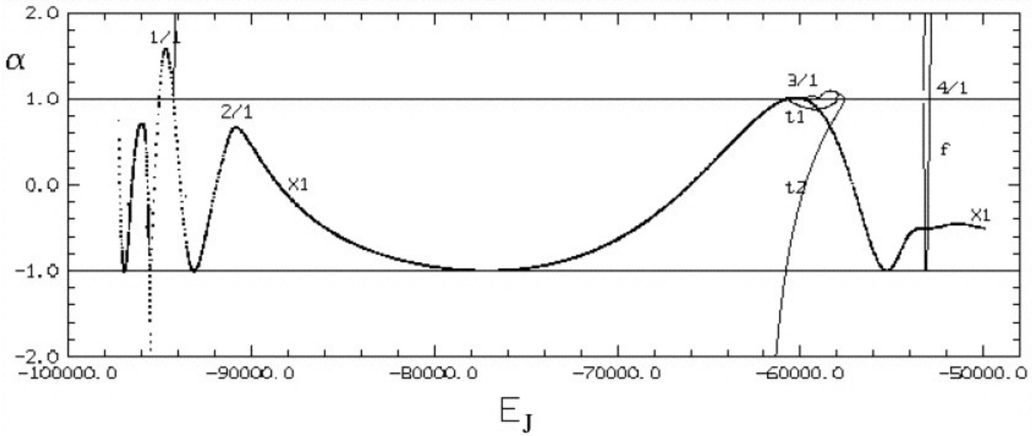



Fig. 4. Stability parameter $\alpha$ for the "central" family of periodic orbits in an axisymmetric galactic potential as a function of radius.

COROTATION

Chaotic zone




## 2nd tool: The stability diagram

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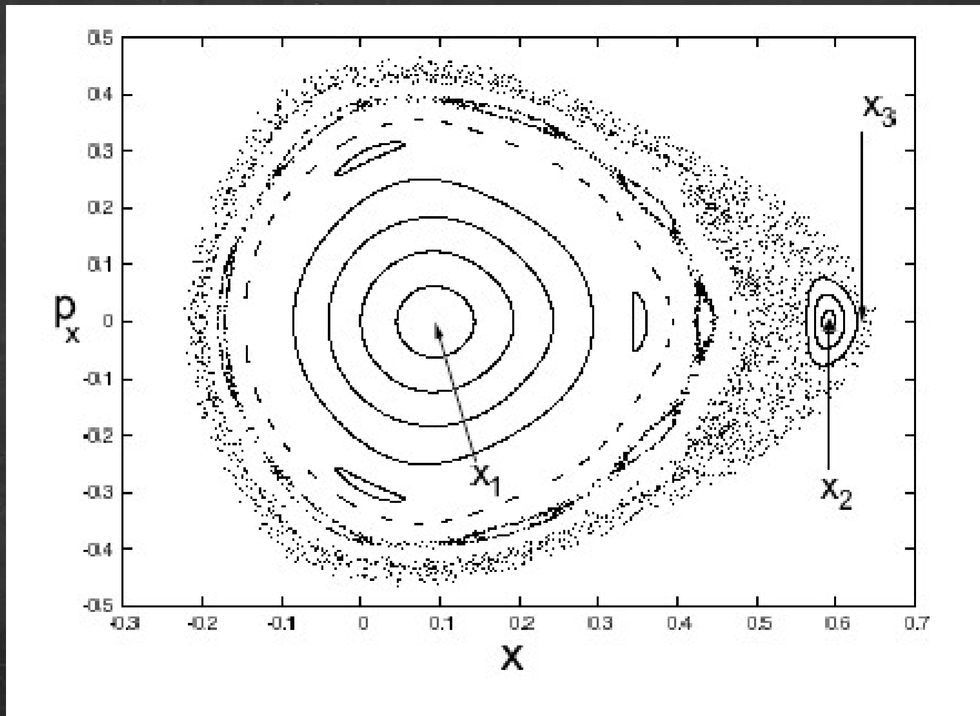
# Rotating $\Phi=\Phi_0+\Phi_b$

At even resonances we have gaps in the characteristics and "horizontal parts" in the stability curves.

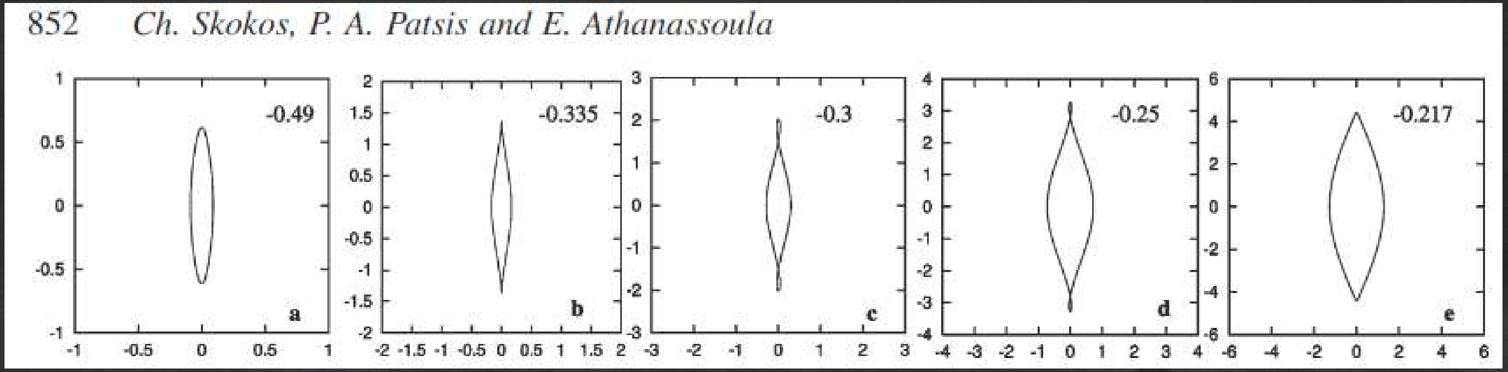
At odd resonances we have "unstable segments" in the characteristics and tangencies or intersections with the $|\alpha|$ axes.

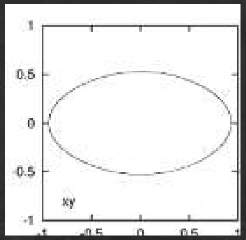
At resonances new families are introduced in the system (bifurcations).

# 3rd . Poincare surfaces of section



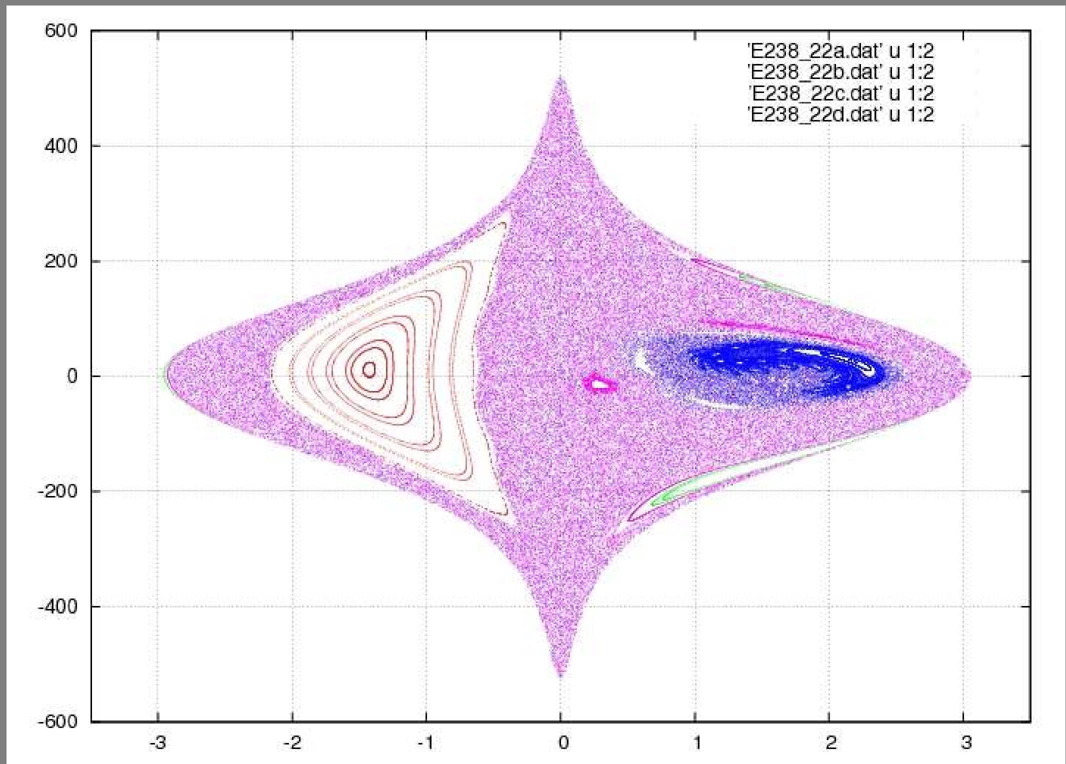
# X1, x2, p.o.


852 Ch. Skokos, P. A. Patsis and E. Athanassoula
-0.49
a
-0.335
b
-0.3
c
-0.25
d
-0.217
e


xy

Below L1,2 (inside corotation).
Chaos+ "Sticky" regions
The bar cannot exceed corotation

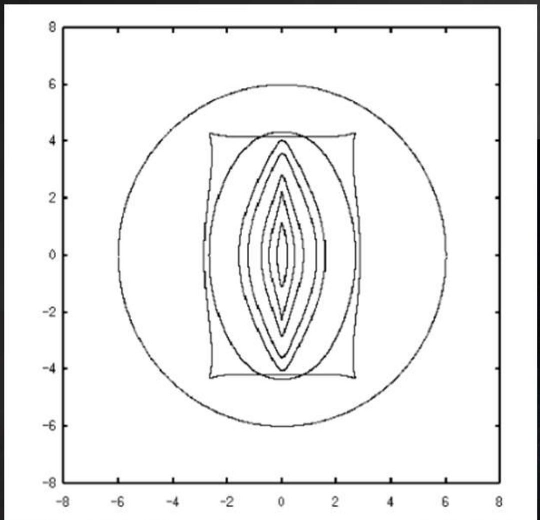
ẋ

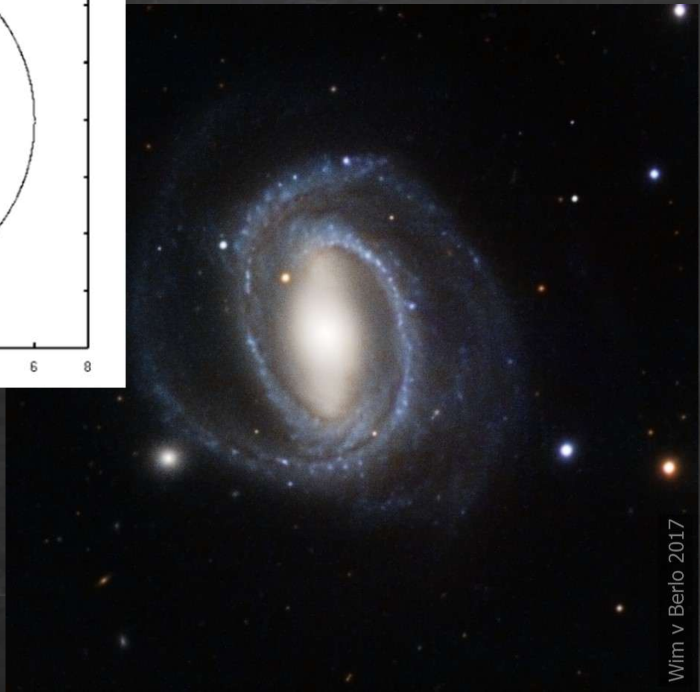


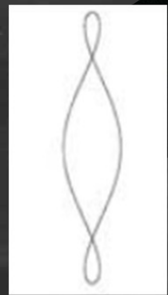

x

The initial idea:

# Stability supports structure



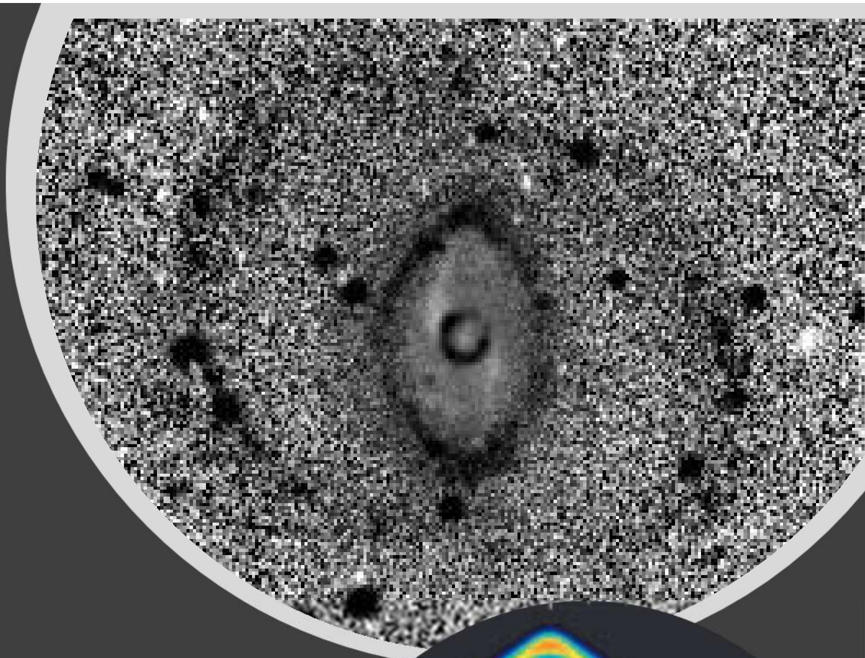



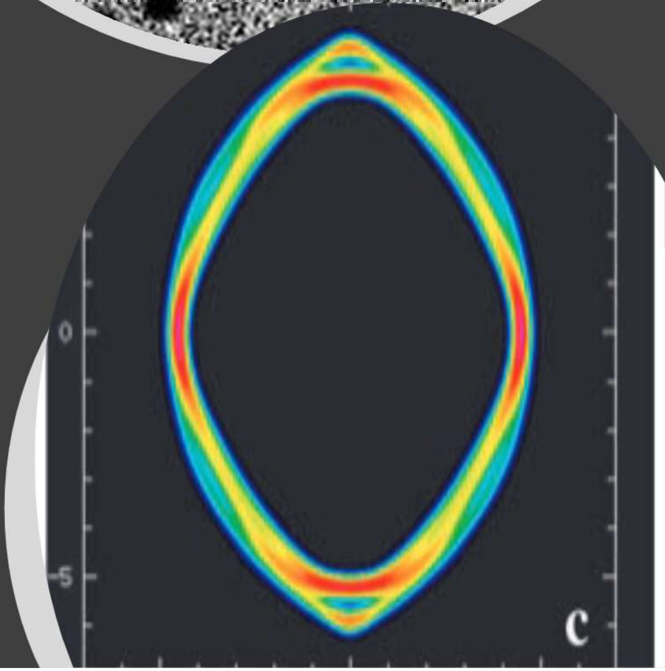



7/16/2025

# Stability supports structure

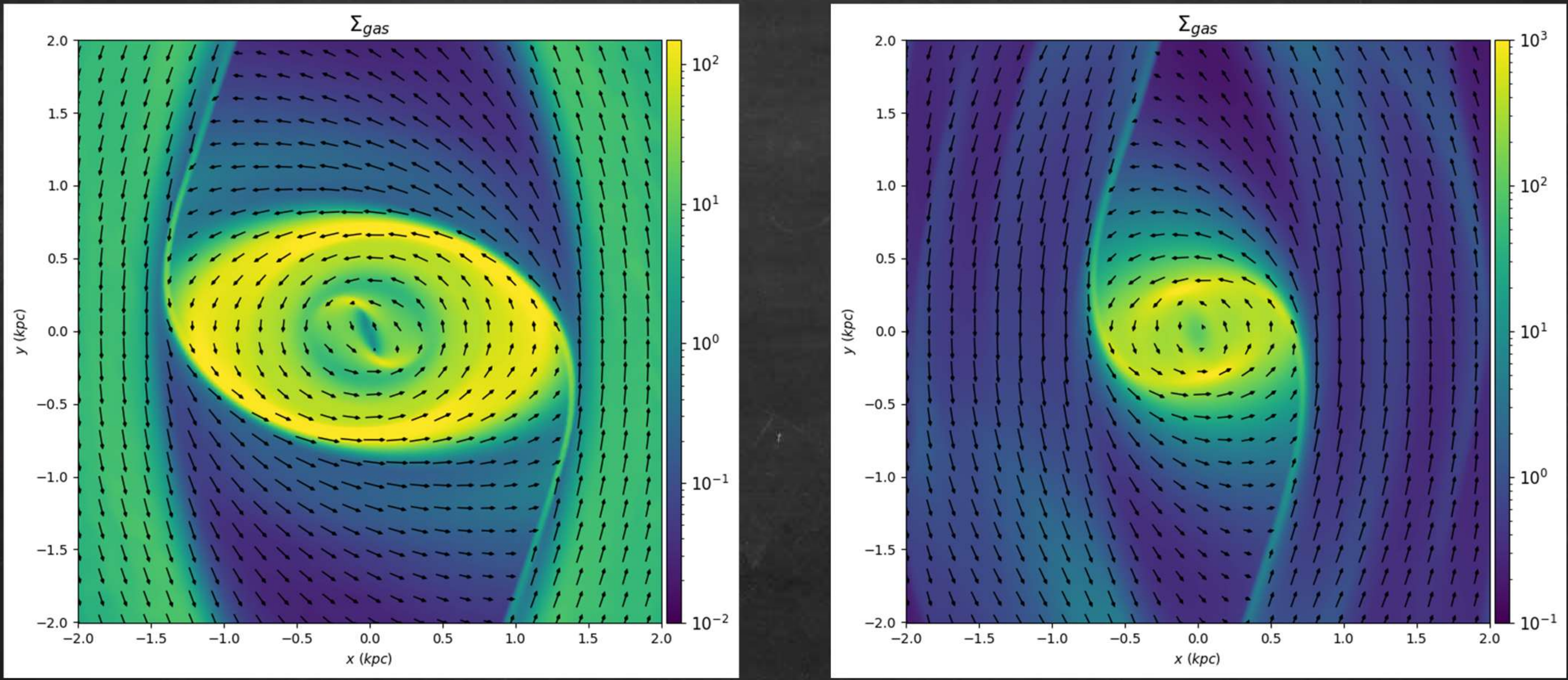
- The stable orbits of the x1 family build the bar (Contopoulos, Athanassoula, Pfenniger....+++)
- The bars do not exceed corotation (Contopoulos 1988)





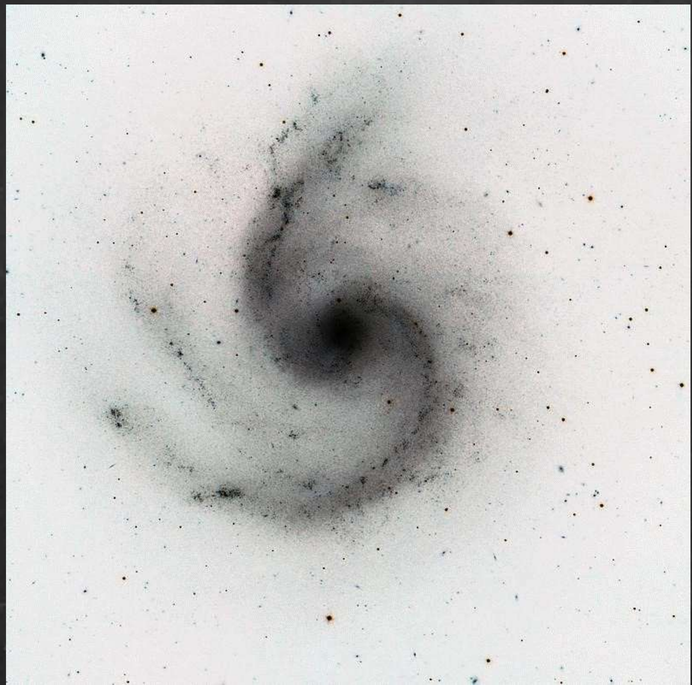
# x2 flows – associated with nuclear rings (Ferrers bar of Athanassoula 1992)

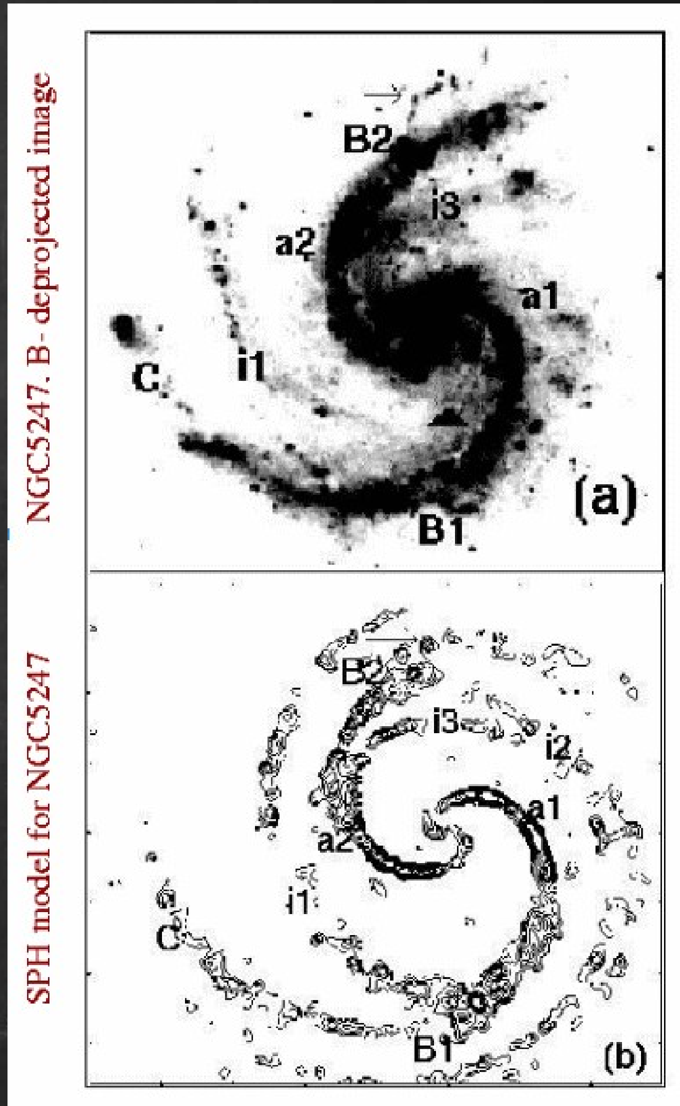
Pastras, Patsis, Athanassoula 2025



Indeed, this assumptions leads to reasonable matchings, also in models for normal (non-barred) spiral galaxies

NGC5247, Hawk-I, Grosbol 2010

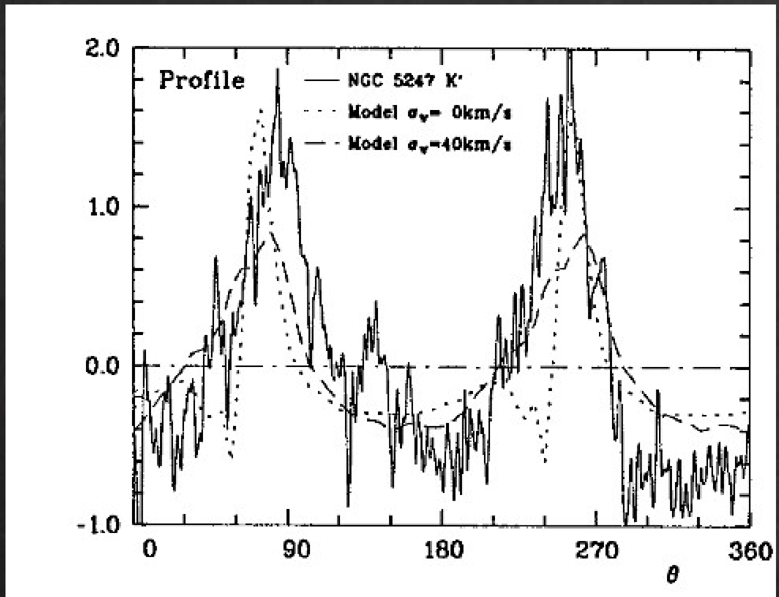




# A, Ωp: Dependence on Hubble type

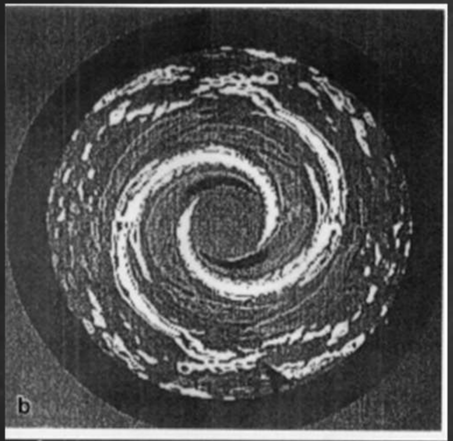
- Ωp ←→ Resonances
- Linear – Non-linear: The strength of the perturbation

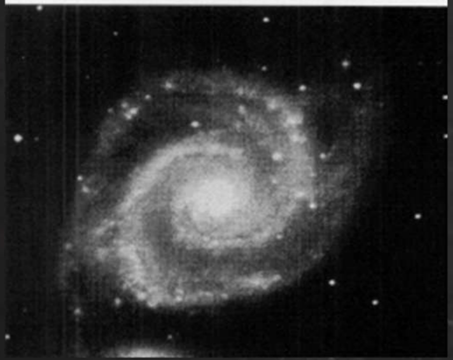
P. Grosbol 1994



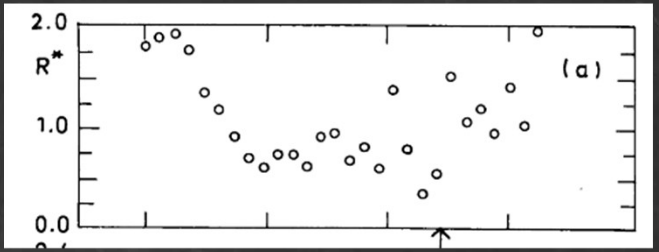

Neither A nor Ωp can be reliably estimated from observations (Population effects, DM, dust etc.) Thus, modelling is needed

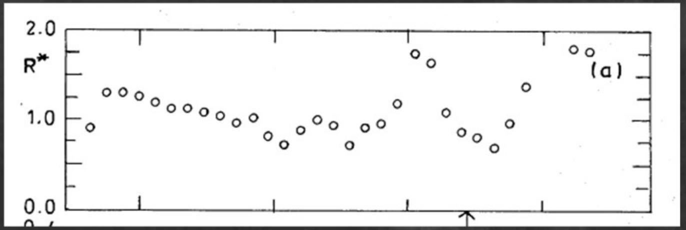
# Self-consistent tests – I. Open spirals

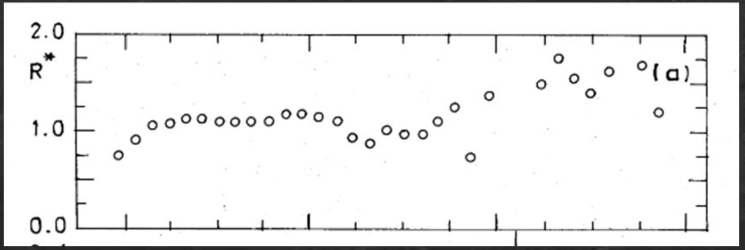


NGC 5194

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# Self-consistent tests – I. Open spirals

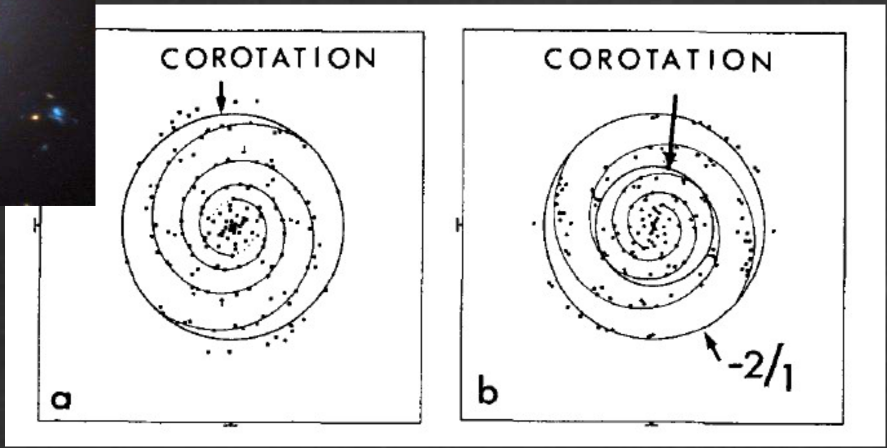
NGC 5194

- Spirals rotate slowly: (symmetric) part ends before corotation
- Off-phase extensions may reach corotation
- A: Fθ/Ftotal 5-12% (strong)

- Basically, quasi-periodic orbits trapped around x1 orbits (compatible with σ – dispersion of velocities)

# Self-consistent tests – II. Tightly wound spirals

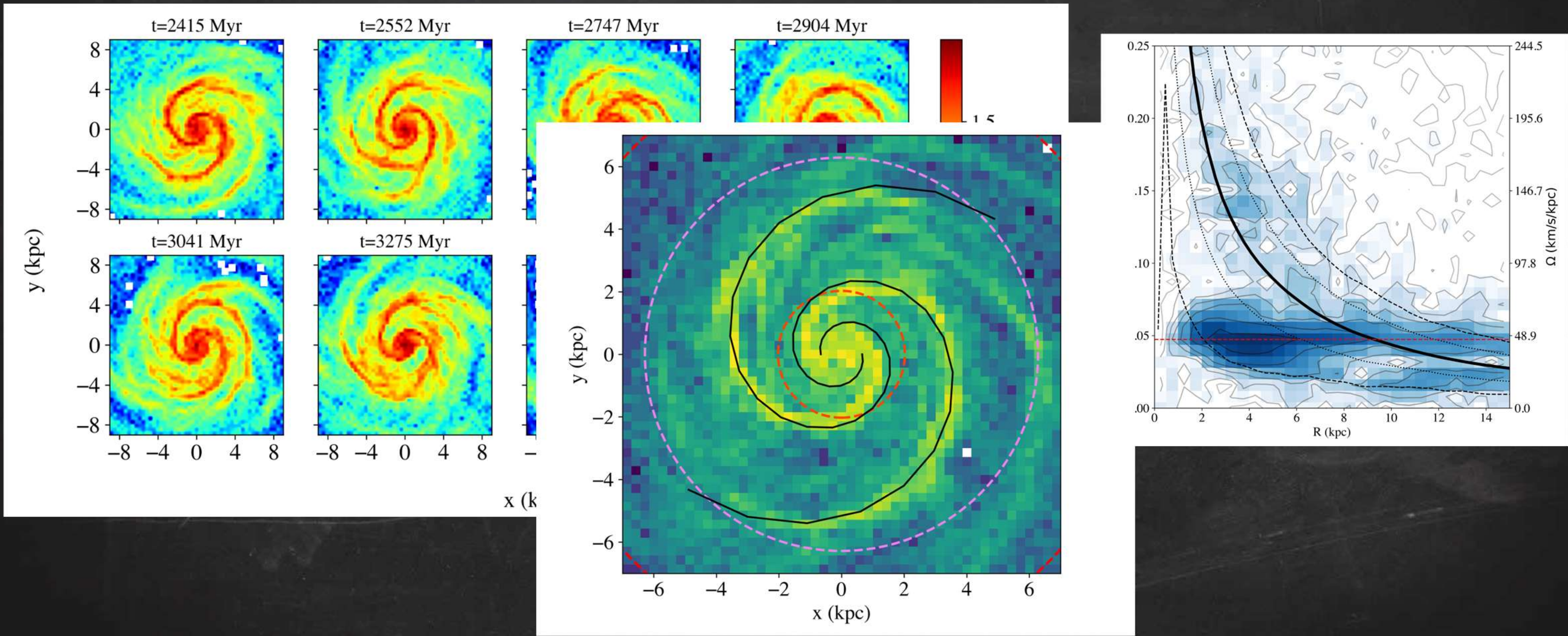
NGC 488 - dprj




- Spirals rotate fast: arms reach/cross corotation
- Off-phase extensions may reach corotation
- A: Fθ/Ftotal about 1% (weak)

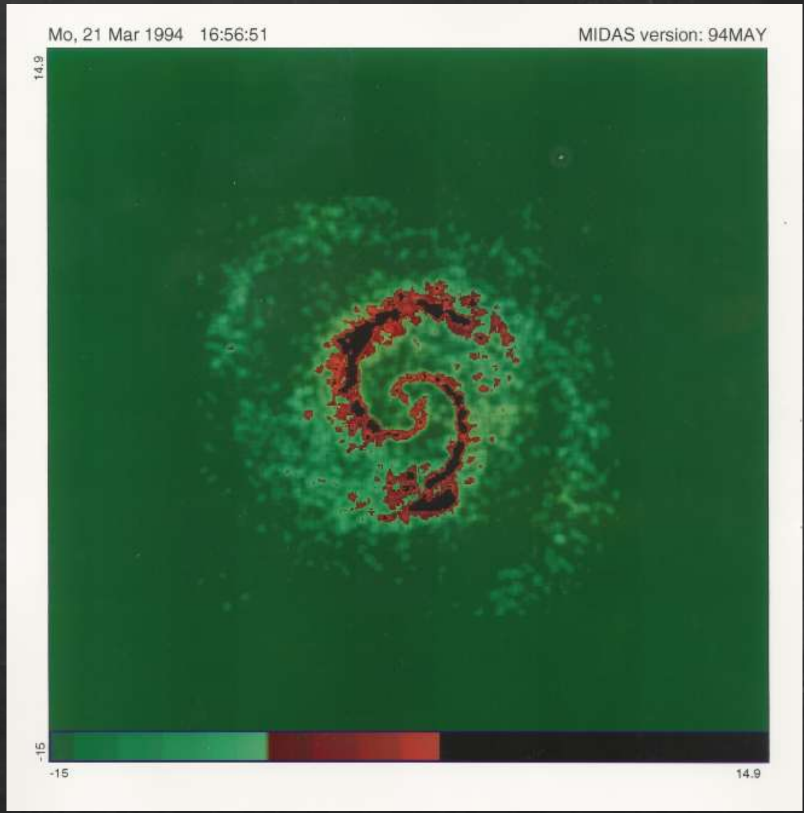
- Basically, quasi-periodic orbits trapped around x1 orbits (compatible with σ – dispersion of velocities)

# ...in preparation (Patsis & Okalidis 2025)



# Beyond the 4/1 resonance

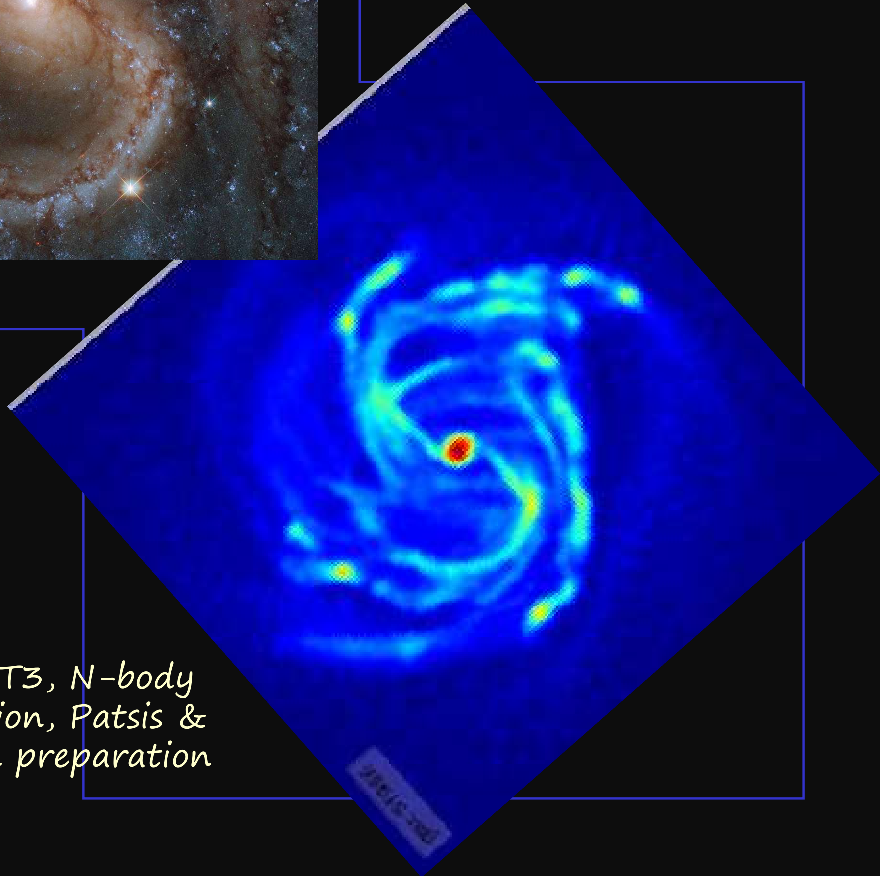
## change of paradigm?



The region beyond corotation is better studied in the case of barred galaxies

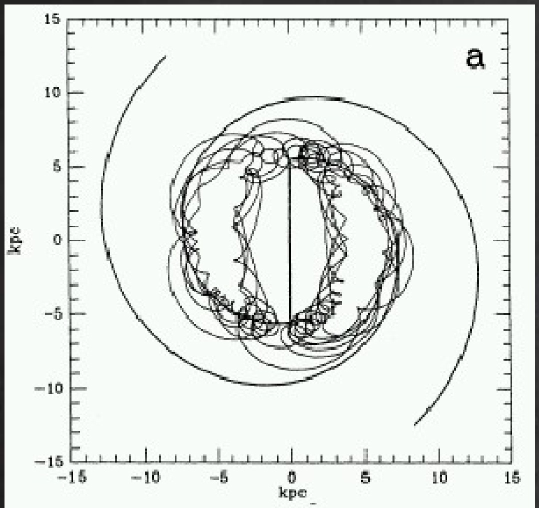
...provided you have one pattern speed $\Omega_b = \Omega_s$

NGC4535, HST

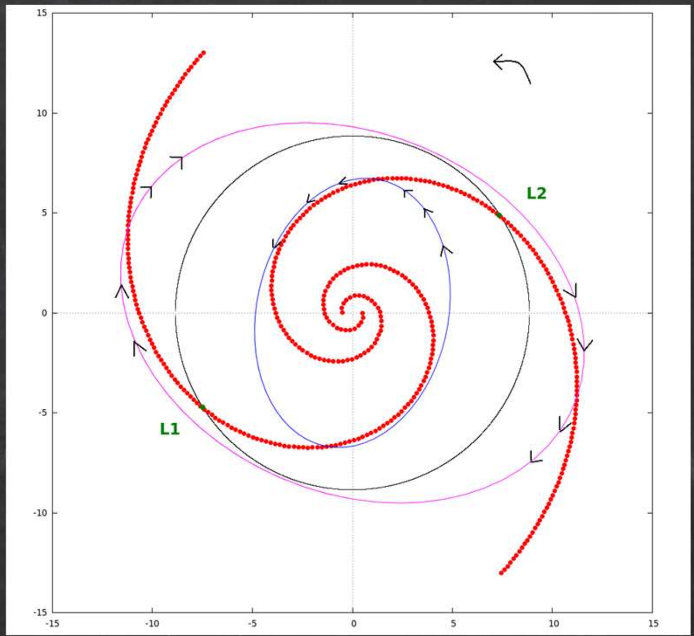


GADGET3, N-body simulation, Patsis & Naab in preparation

# "Chaotic" spirals. The "grey" zone and beyond




Kaufmann & Contopoulos 1996



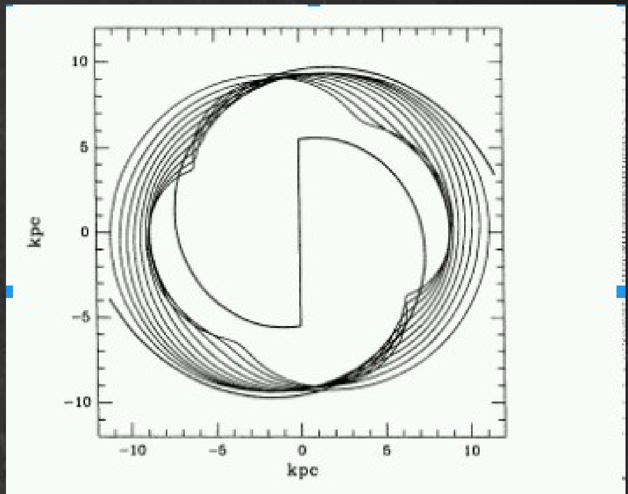
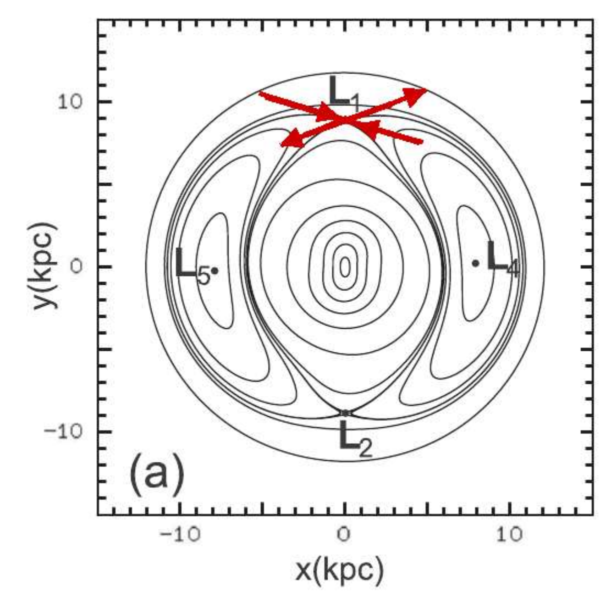
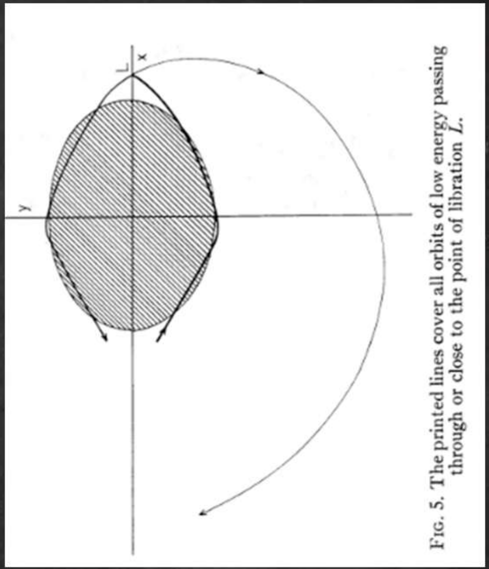




**Fig. 19.** The −2/1 family of periodic orbits in the best model of NGC 3992. Also shown are the minima of the bar and spiral potentials

# …later, "Chaotic" spirals, Lyapunov orbits, etc.

- Patsis 2006, MNRAS 369, L56
- Romero-Gomez, Masdemont, Athanassoula, 2006, A&A 453, 39
- Voglis, Stavropoulos, Kalapotharakos, 2006, MNRAS 372,901
- Voglis, Tsoutsis, Efthymiopoulos, 2006, MNRAS 373,280
- Romero-Gomez, Athanassoula, Masdemont 2007, A&A 472, 63
- Athanassoula, Romero-Gomez, Masdemont 2009, MNRAS 394,67
- ……

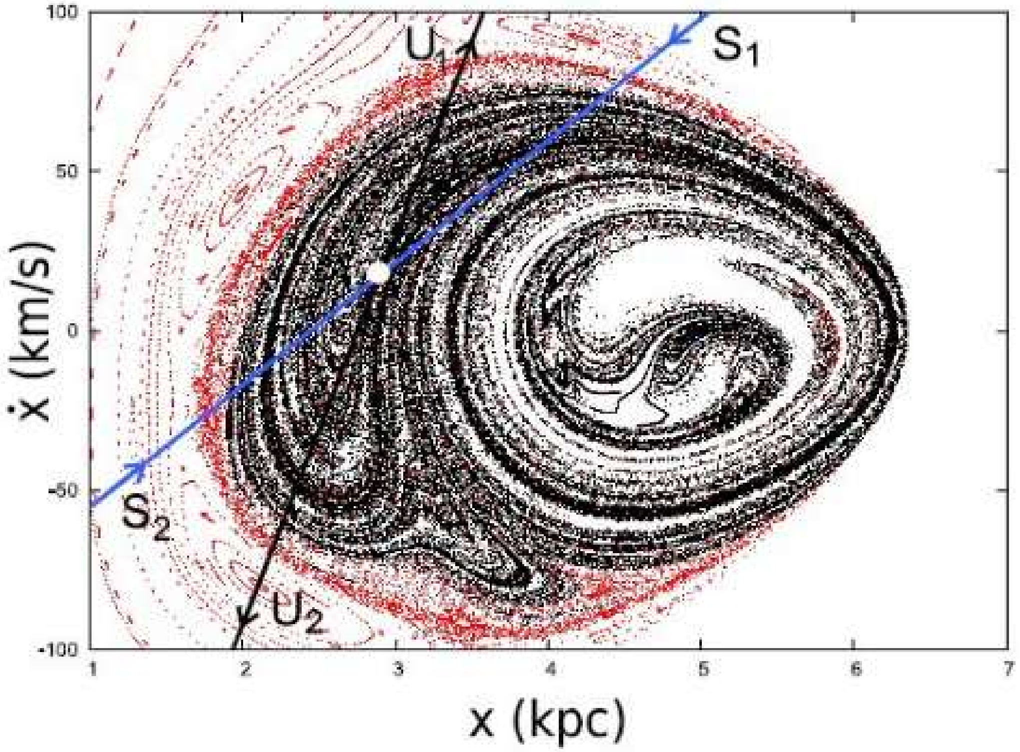



Danby 1965

# Manifolds on Poincare cross sections

Tsigaridi & Patsis 2013




As Poincaré put it: "The reader will be struck by the complexity of this figure, which I am not even attempting to draw. Nothing could give us a better idea of the intricacy of the three-body problem, and of most problems in dynamics."[1]

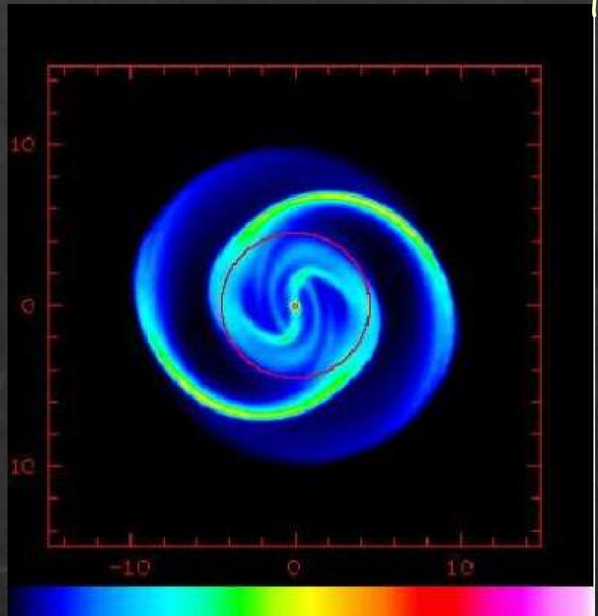
1. *Méthodes nouvelles*, vol. 3, chap. 3, sec. 397.

# The gas flow beyond corotation
# (in any system corotation can be defined)

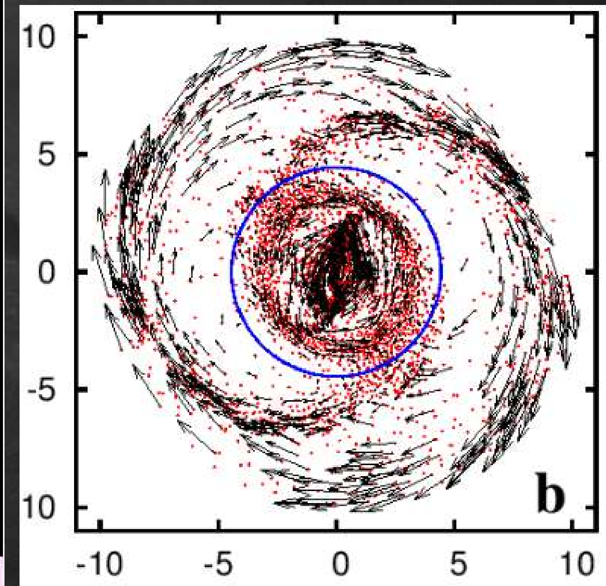
Potentials estimated from NIR images
NGC4314, 3359, 1300, 7479, etc.

$$\Phi(r,\varphi) = \Phi_0(r) + \sum_{m=2,4,6} \Phi_{mc}(r)\cos(m\varphi) + \Phi_{ms}(r)\sin(m\varphi). \quad (1)$$

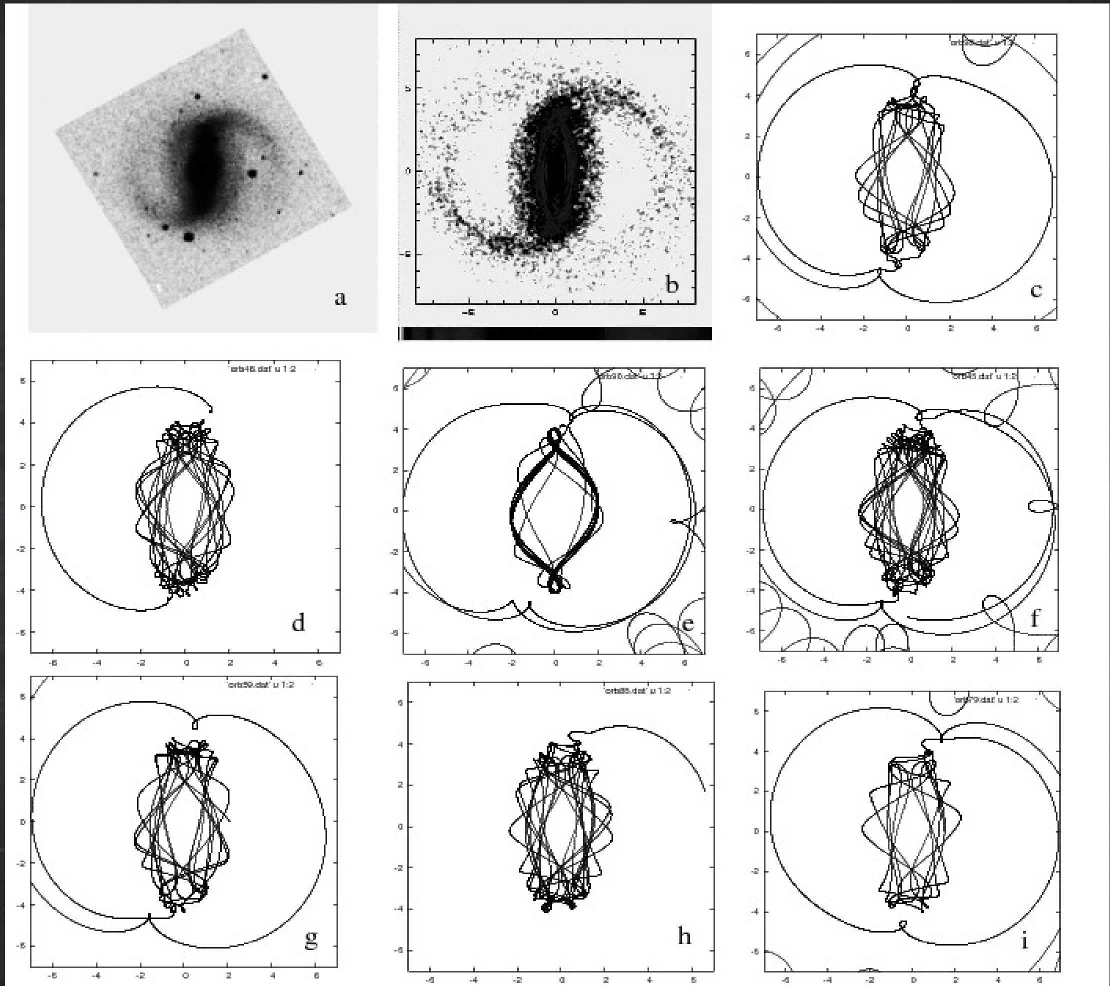
The components $\Phi_0(r)$, $\Phi_{mc}(r)$, and $\Phi_{ms}(r)$ of the equation above are given as polynomials of the form $\sum_n a_n r^n$, $n = 0, \ldots, 8$.



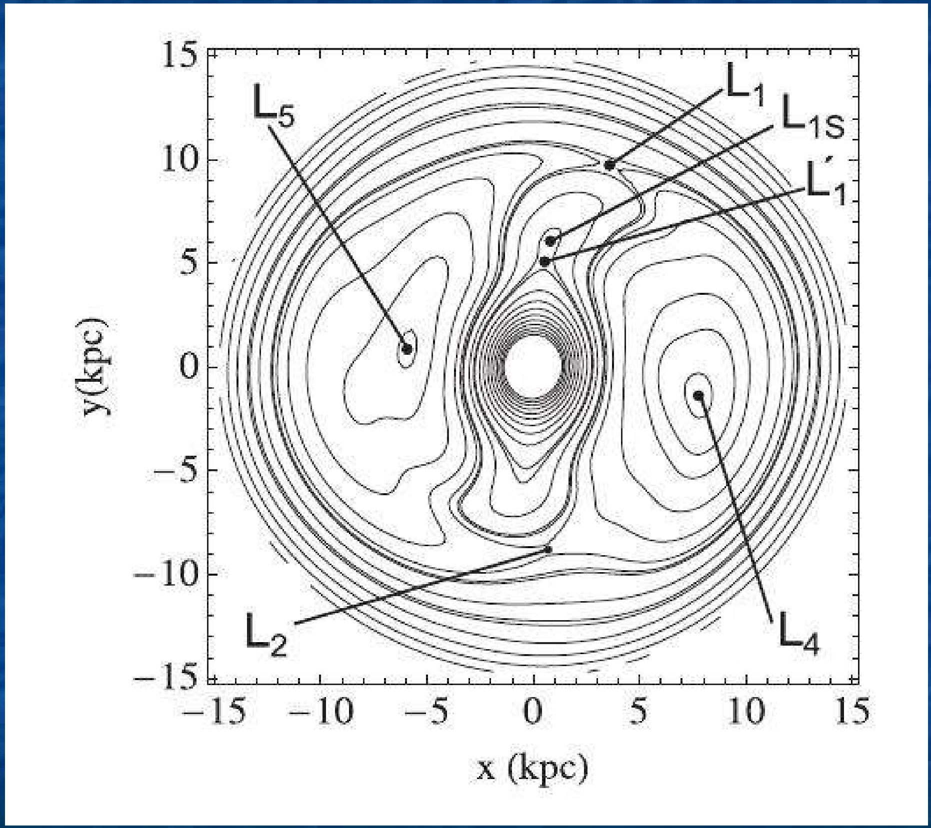

The flow is ALONG the arms




Patsis & Tsigaridi 2017


a
b
c
d
e
f
g
h
i

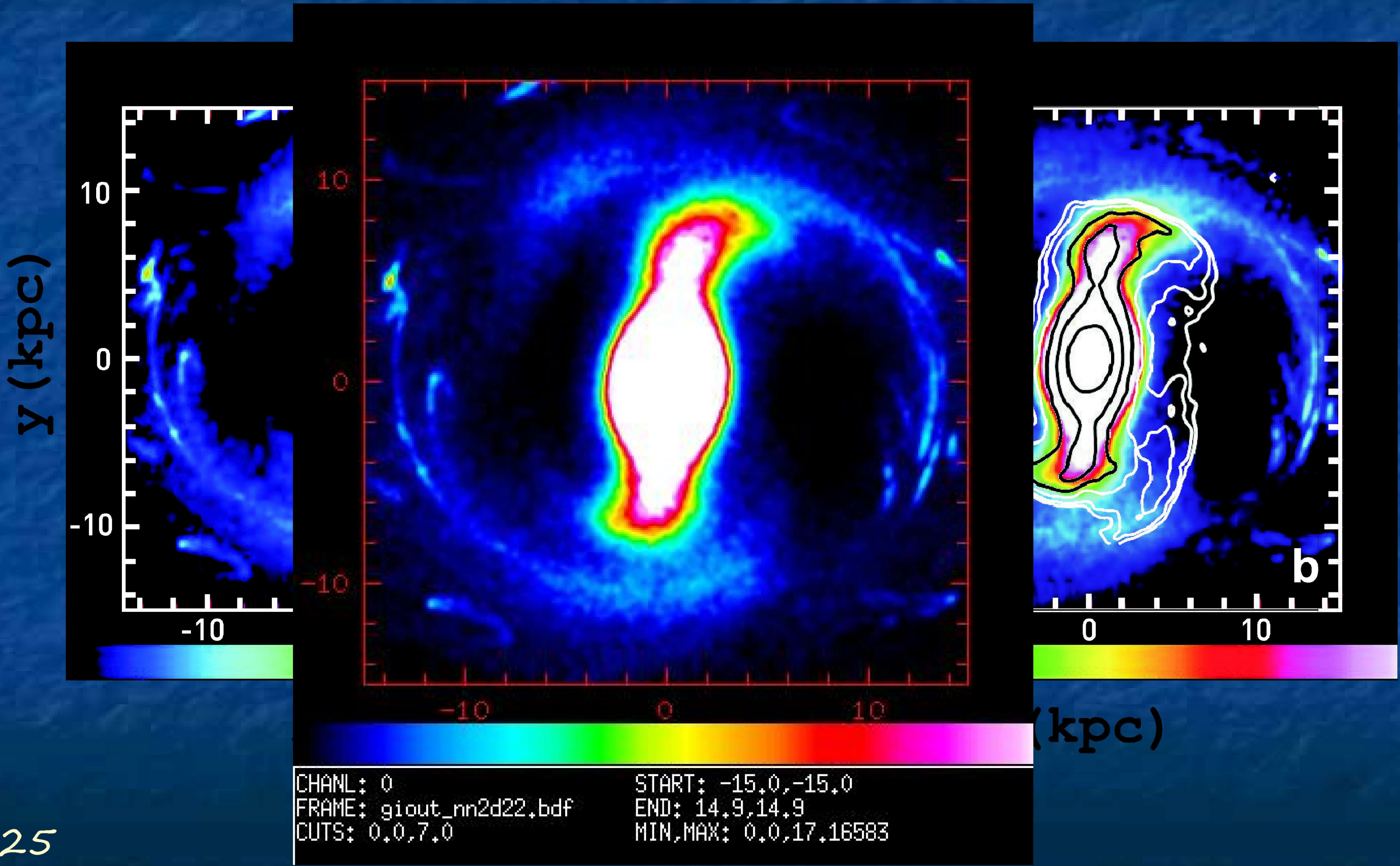
A didactic 2D model of a response of a potential estimated for NGC1300 ($\Omega_p$ = 22 km s$^{-1}$ )



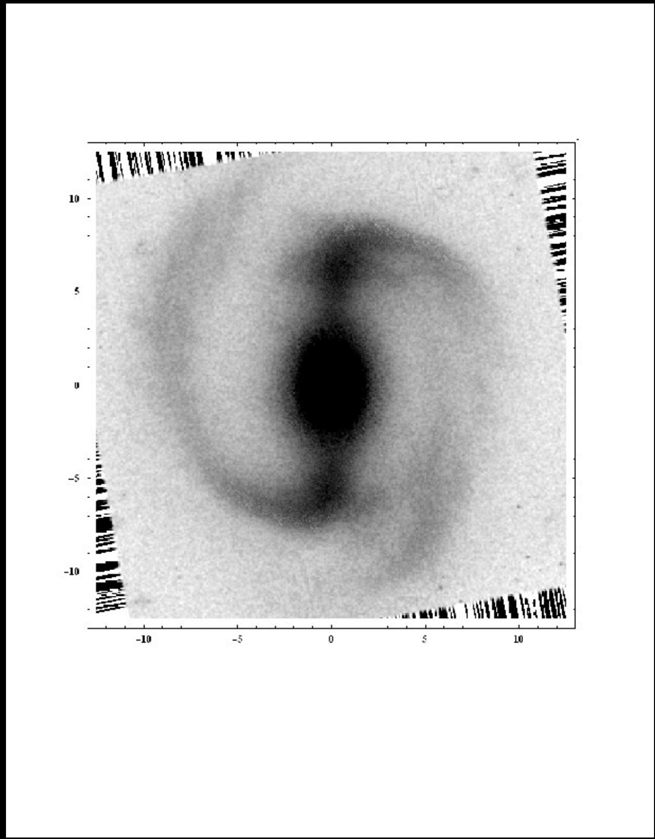

7/16/2025

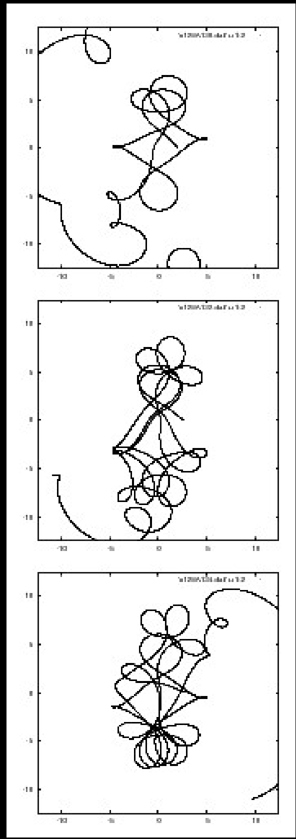
The 2D response –

# Structure out of Chaos



# Chaotic orbits shaping a bar





# 3D systems

$\Phi$=Miyamoto disk + Plummmer sphere + 3D Ferrers bar

$$H = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) + \Phi(x, y, z) - \Omega_b(xp_y - yp_x),$$

with

$$\Phi(x, y, z)_{eff} = \Phi(x, y, z) - \Omega_b(xp_y - yp_x)$$

$$\dot{x} = p_x + \Omega_b y, \quad \dot{y} = p_y - \Omega_b x, \quad \dot{z} = p_z$$
$$\dot{p}_x = -\frac{\partial \Phi}{\partial x} + \Omega_b p_y, \quad \dot{p}_y = -\frac{\partial \Phi}{\partial y} - \Omega_b p_x, \quad \dot{p}_z = -\frac{\partial \Phi}{\partial z}$$

$$\Phi(x, y, z) = \Phi_D + \Phi_S + \Phi_B$$

4D spaces of section, i.c. $(x, p_x, z, p_z)$ in the plane $y=0$ with $p_y>0$

# Linear Stability of p.o. in 3D systems

The relation of the final deviations of this neighboring orbit from the periodic one, with the initially introduced deviations can be written in vector form as: $\vec{\xi} = M\vec{\xi}_0$. Here $\vec{\xi}$ is the final deviation, $\vec{\xi}_0$ is the initial deviation and $M$ is a $4 \times 4$ matrix, called the monodromy matrix. It can be shown that the characteristic equation is written in the form $\lambda^4 + \alpha\lambda^3 + \beta\lambda^2 + \alpha\lambda + 1 = 0$. Its solutions $(\lambda_i, i = 1, 2, 3, 4)$ obey the relations $\lambda_1 \lambda_2 = 1$ and $\lambda_3 \lambda_4 = 1$ and for each pair we can write:

$$\lambda_i, 1/\lambda_i = \frac{1}{2}[-b_i \pm (b_i^2 - 4)^{\frac{1}{2}}],$$

where $b_i = 1/2\,(\alpha \pm \Delta^{1/2})$ and stability indices

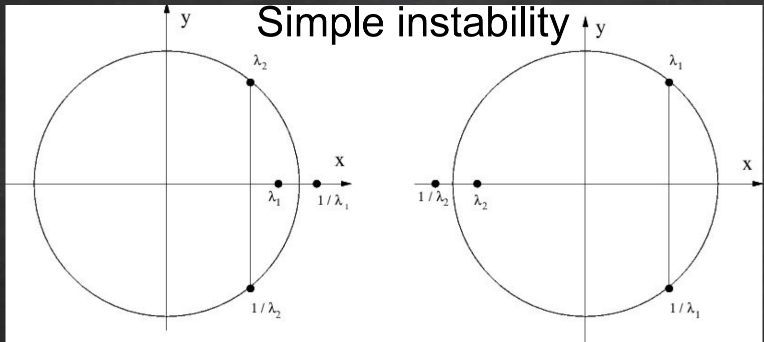
$$\boxed{\Delta = \alpha^2 - 4(\beta - 2).}$$

motion is stable when all the roots of (44) are complex conjugate lying on the unit circle, and this happens when the following three inequalities hold:

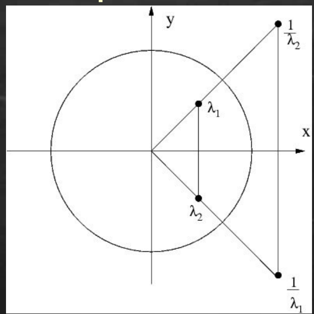
$$\Delta > 0, \qquad |b_1| < 2, \qquad |b_2| < 2. \tag{49}$$

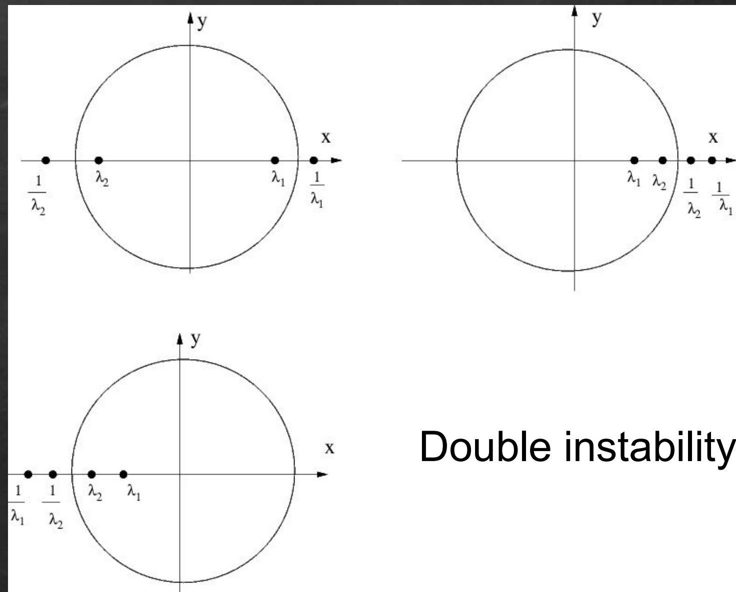
In all other cases the motion is unstable.

Simple instability




Complex instability



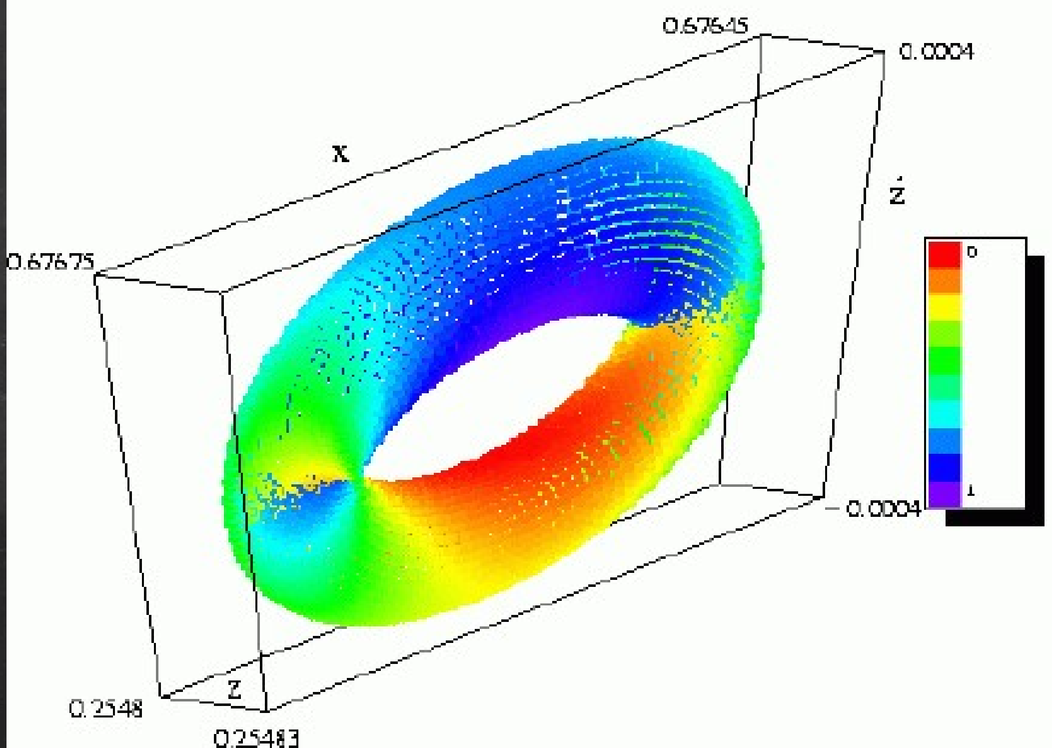

Double instability

Katsanikas et al. 2011-13, Int. J. Bif. Chaos

Color-rotation method
Patsis & Zachilas 1994,
IJBC 4

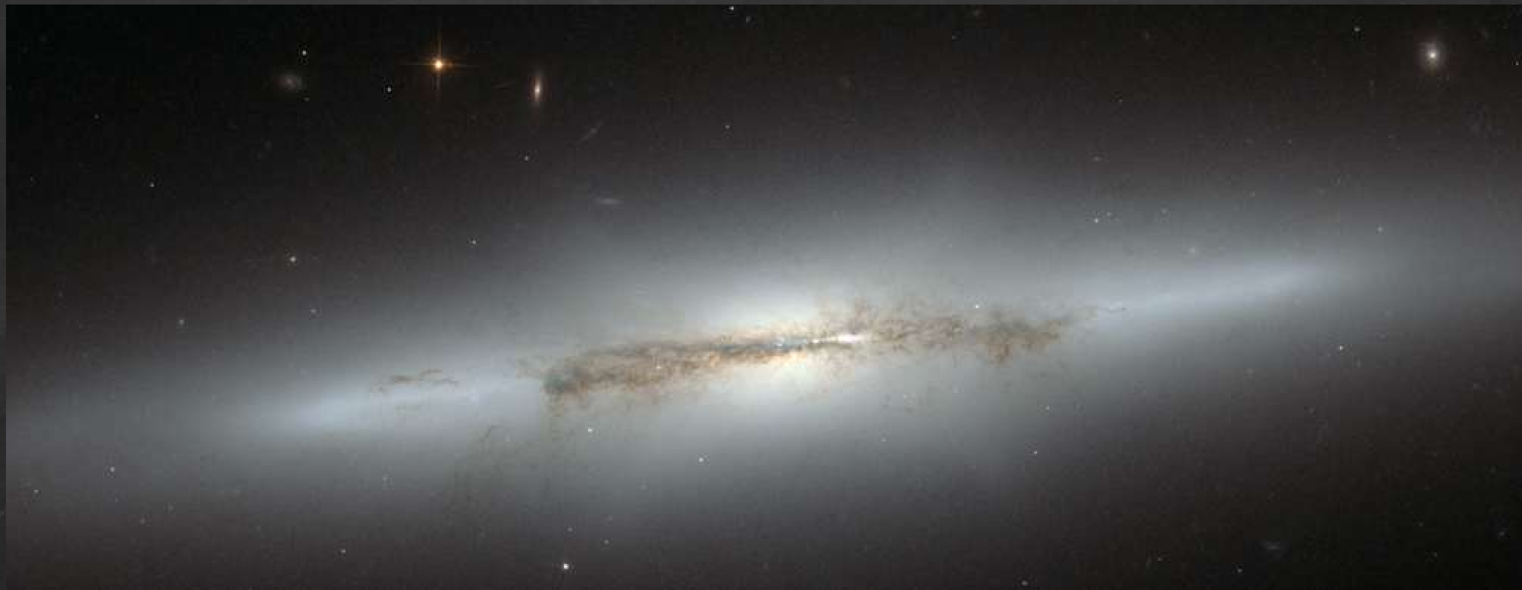
The structure of phase space
in the neighborhood of periodic orbits

Stability



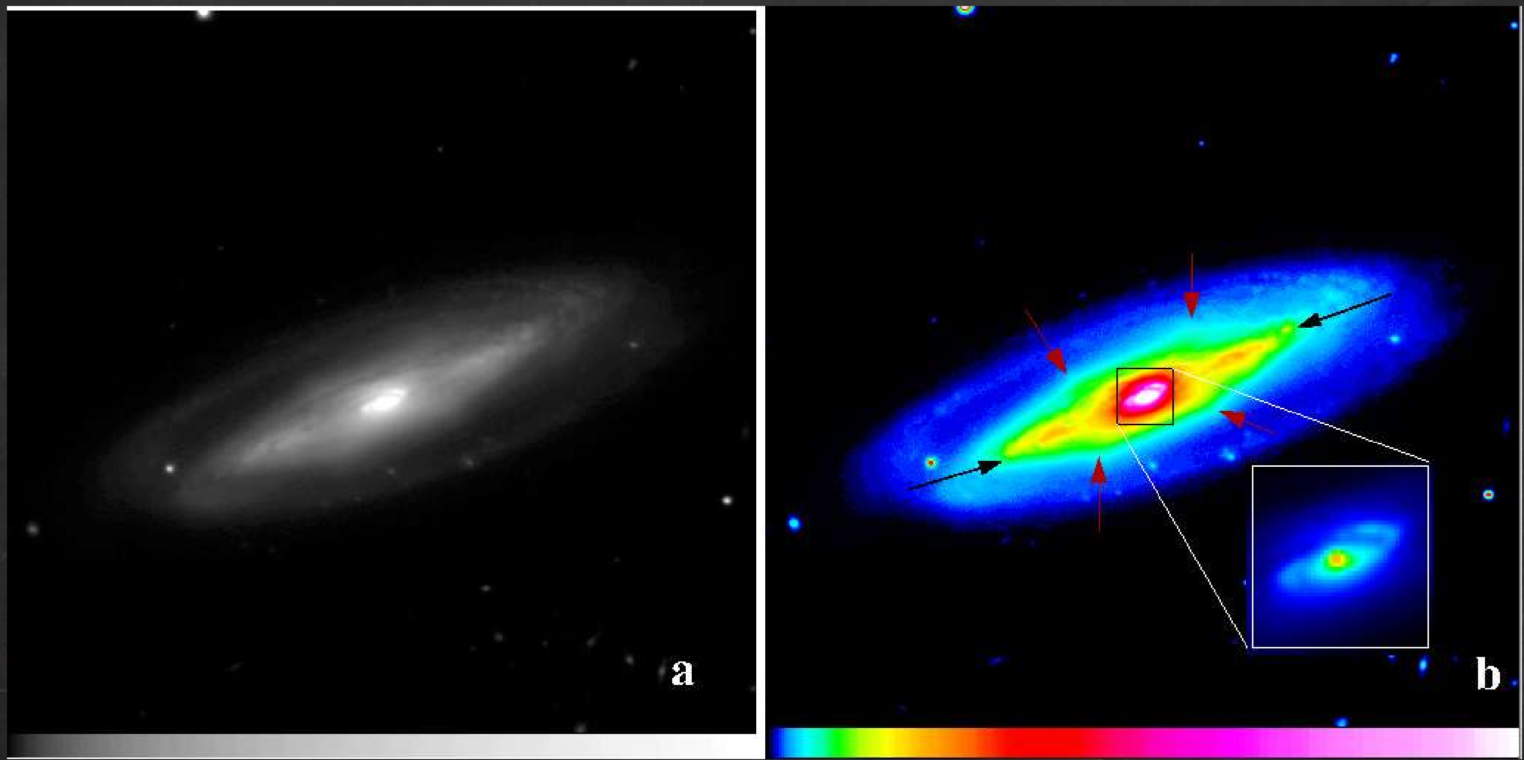
# Boxy/peanut bulges and "X"

NGC 4710, $\alpha=12^h\ 49^m\ 38.9$ , $\delta=+15°\ 9'\ 56''$



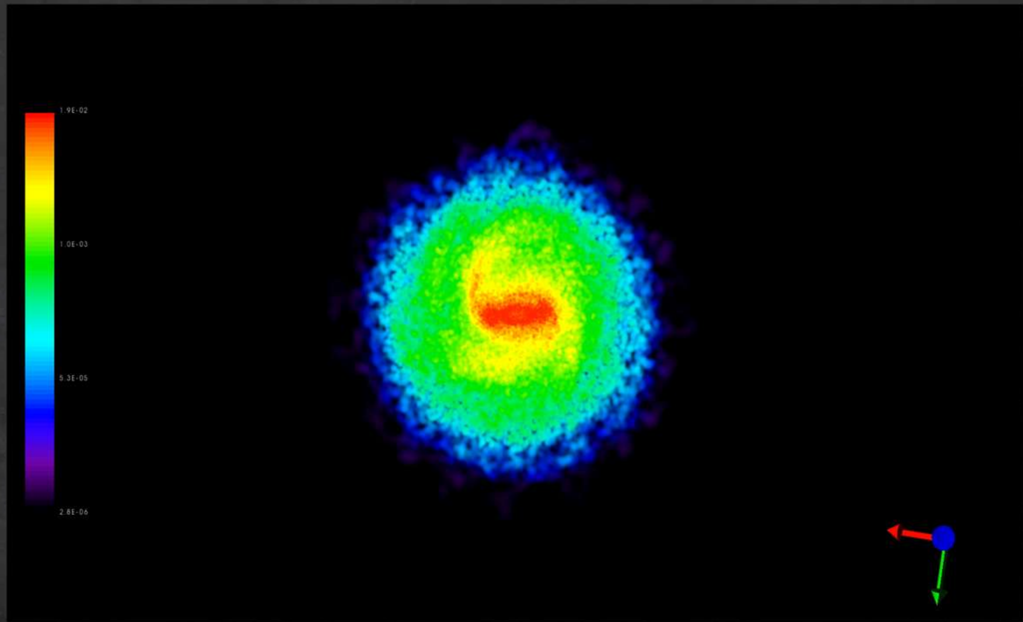
This natural-color photo was taken with the Hubble Space Telescope's Advanced Camera for Surveys on January 15, 2006

# NGC352 (Aristarchos telescope, Helmos, Greece)

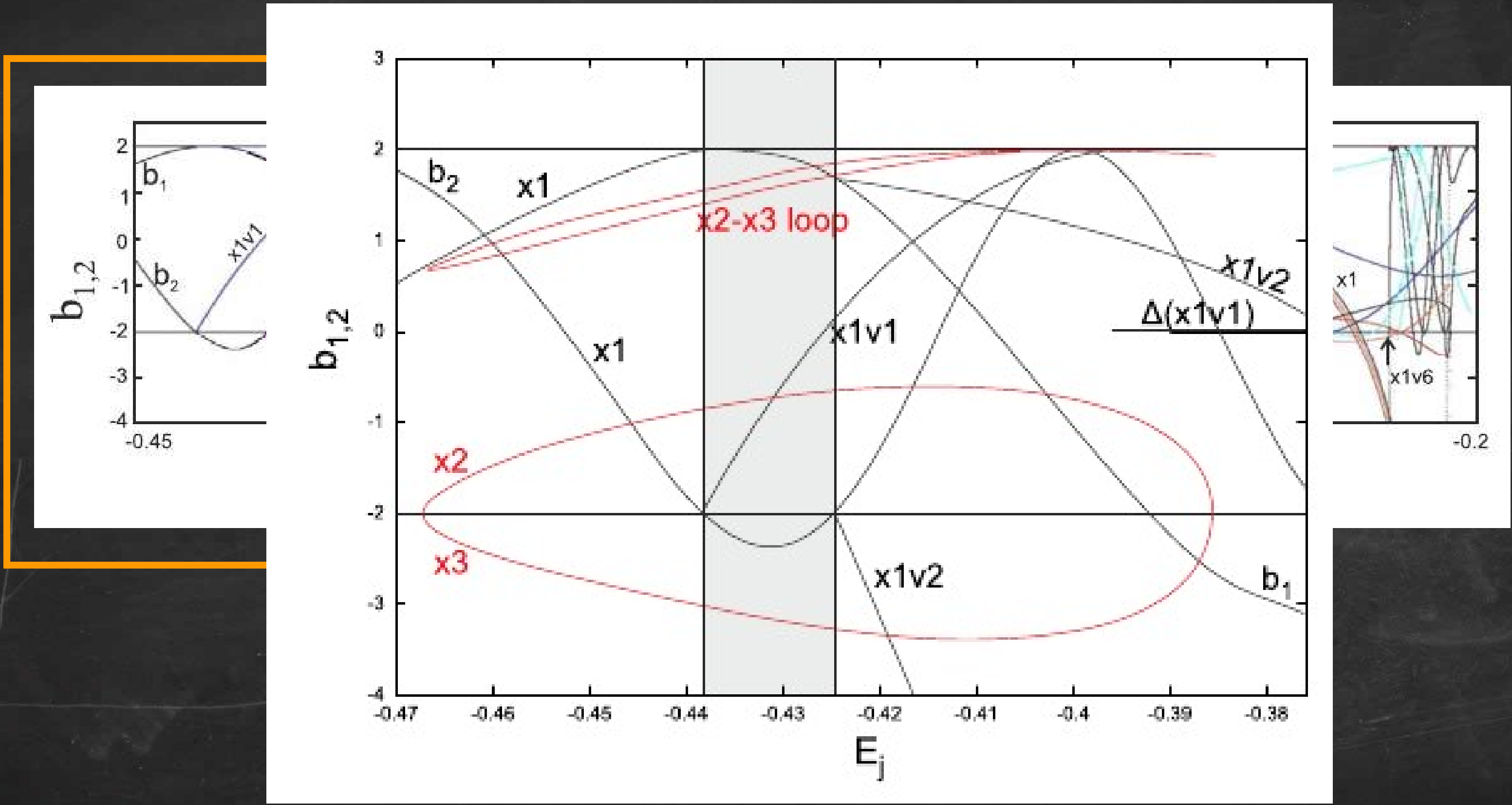



R filter. Patsis, Xilouris, Alikakos 2021

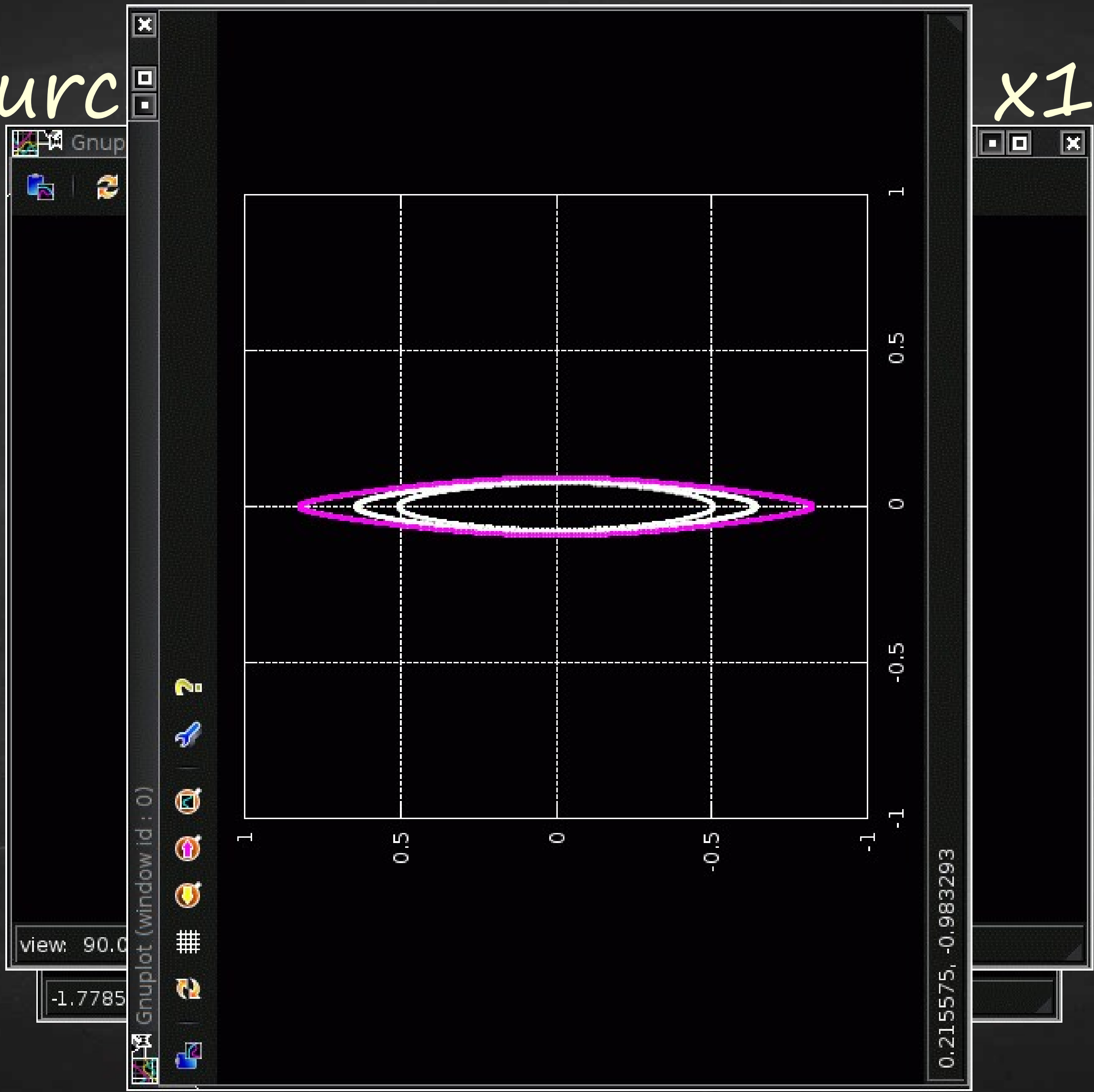
# Traveling around the galaxy...



# 1. Where does the b/p start?

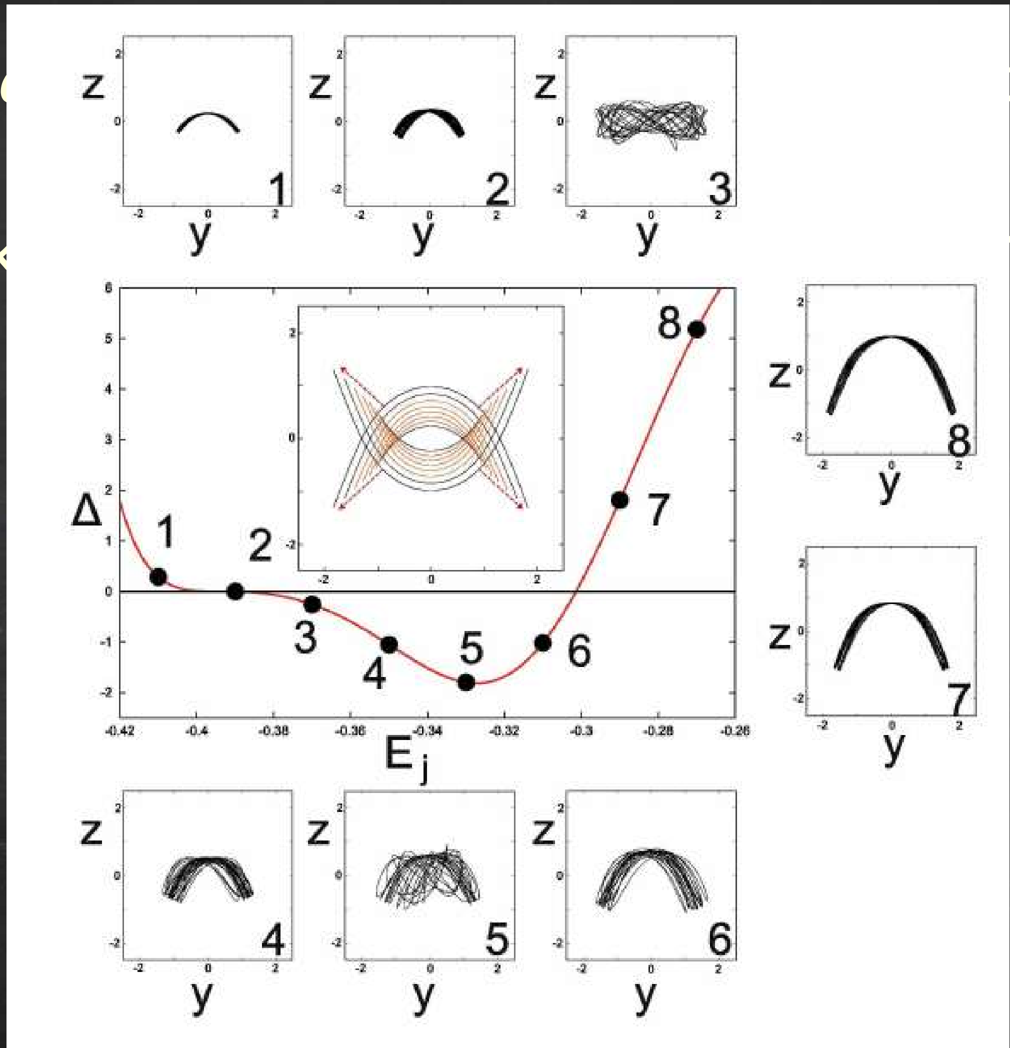


# Bifurc x1v1'

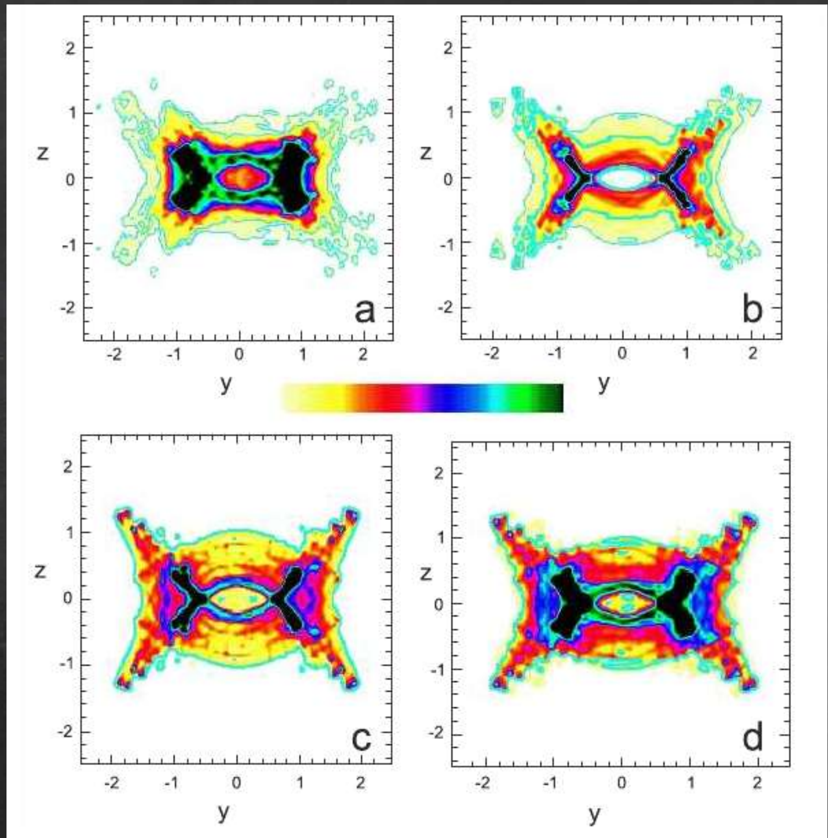

Gnup
view: 90.0
-1.7785
Gnuplot (window id : 0)
0.215575, -0.983293
1
0.5
0
-0.5
-1

## 2. Ass… of p.o.

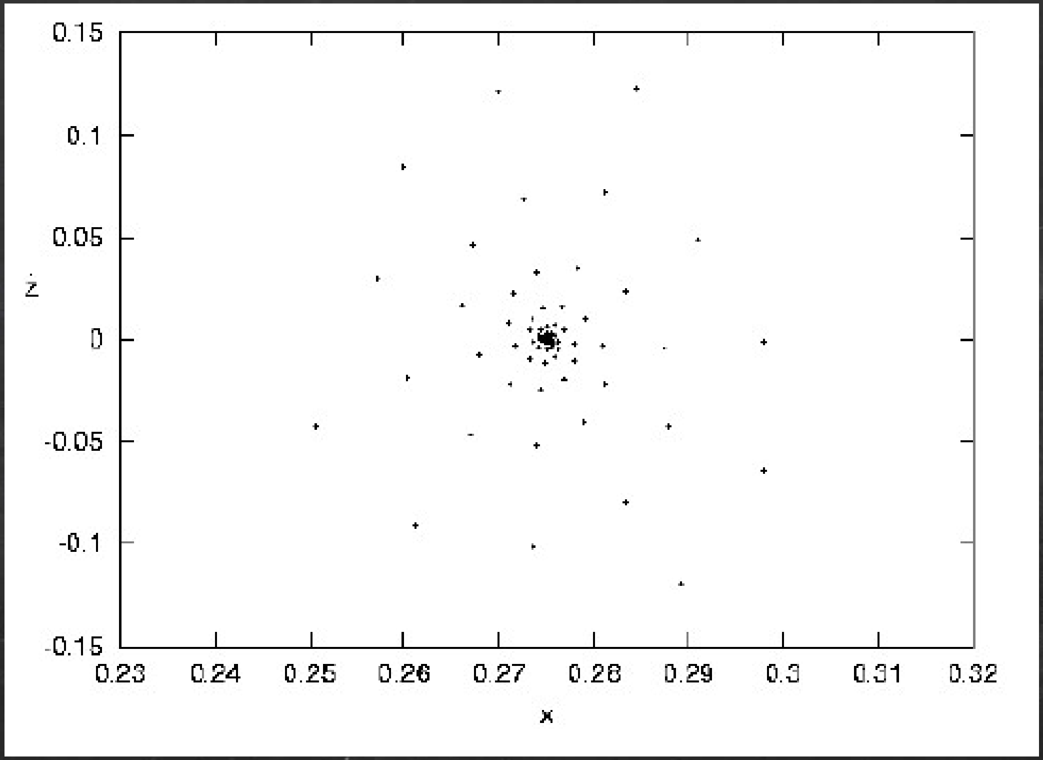
- X1, x… ram)



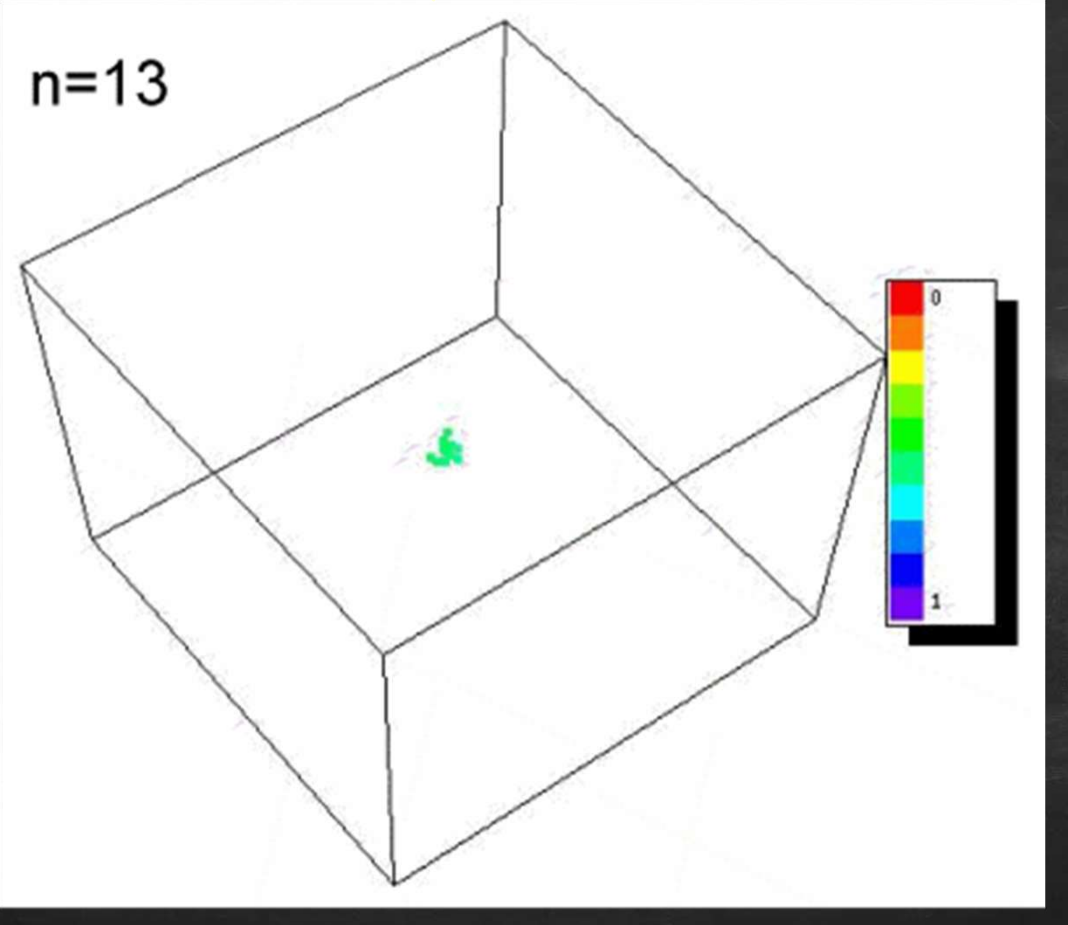
# the "x1v1" scenario



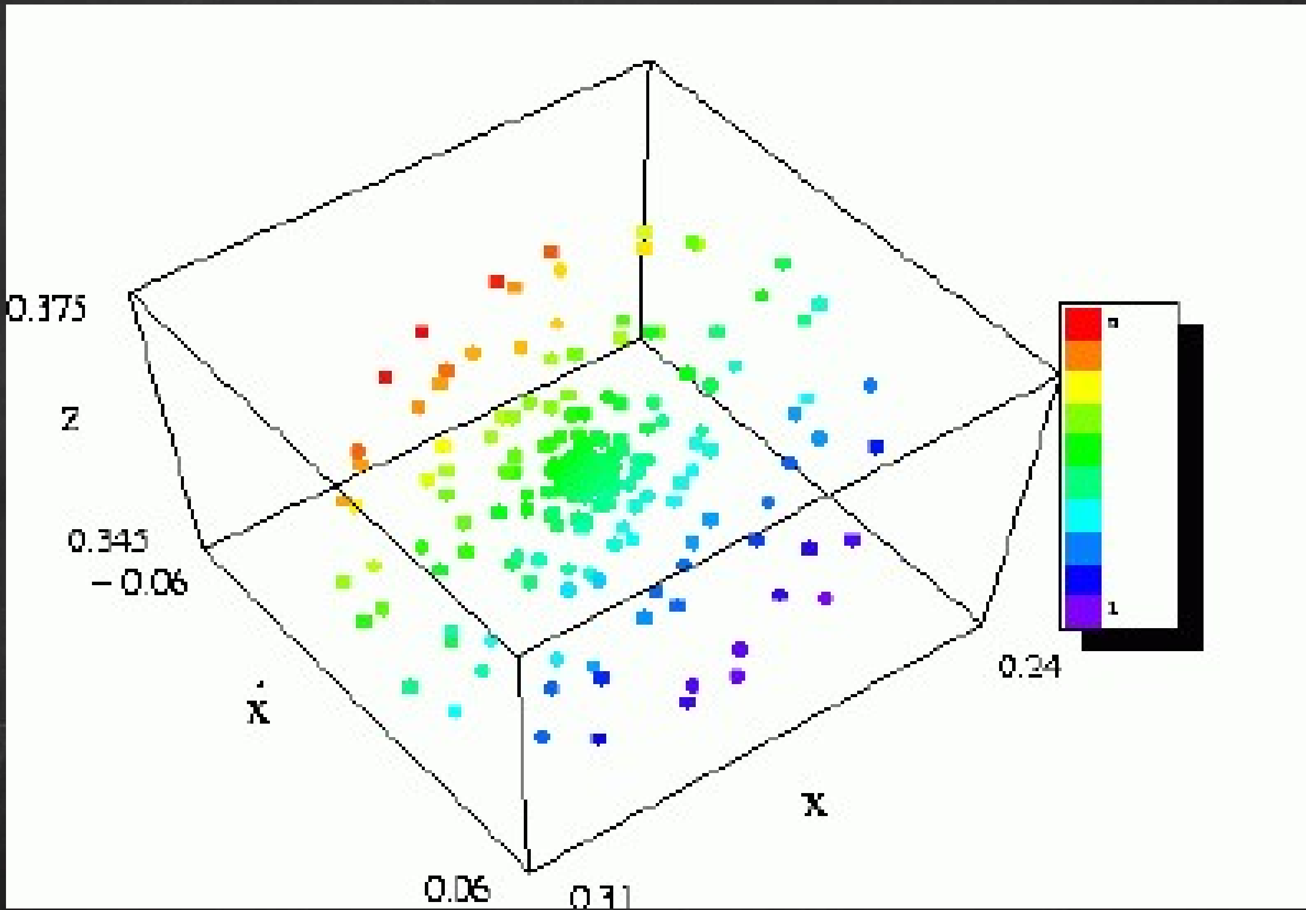
# Complex Instability: 2D projections of the 4D s.o.s. (Contopoulos, Farantos, Papadaki, Polymilis 1994)



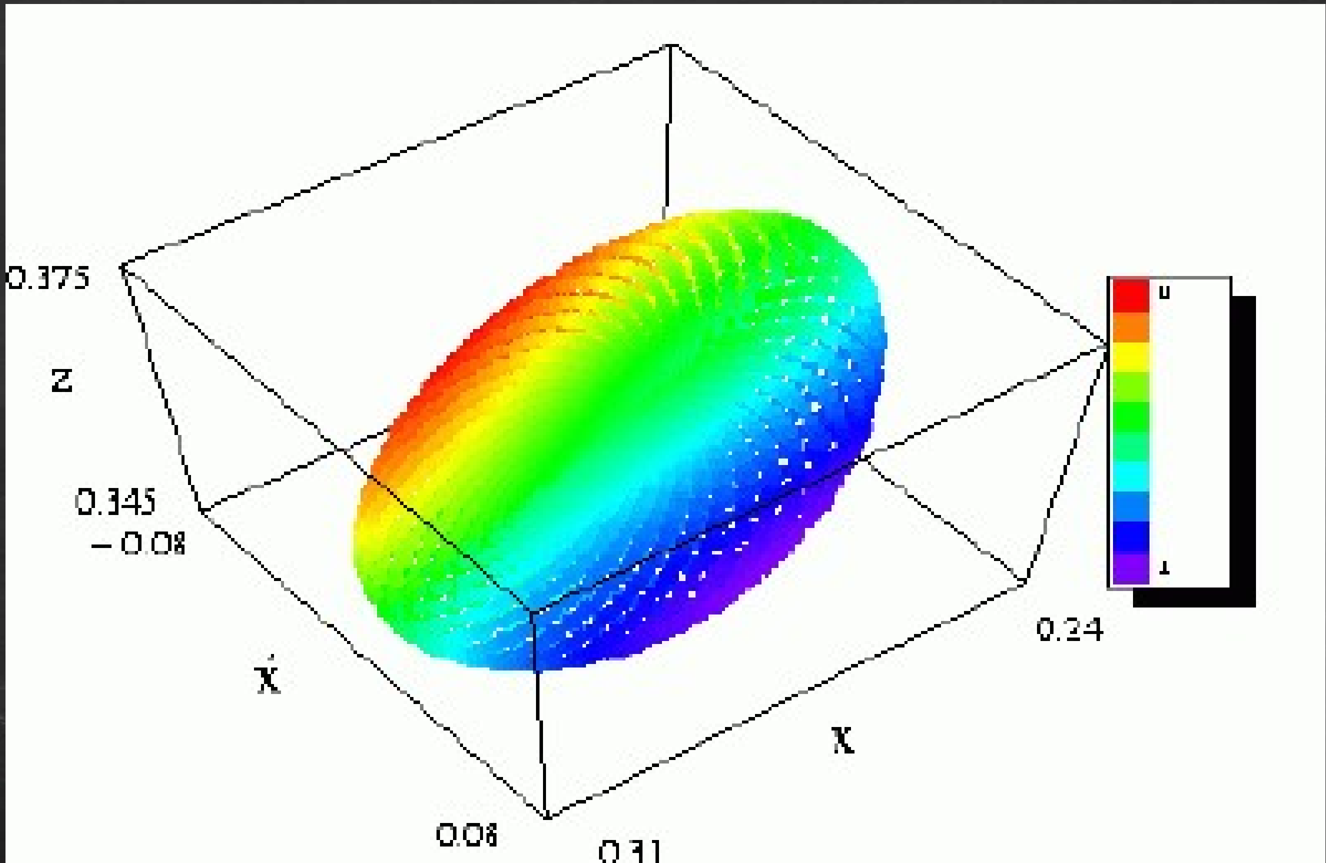
# Complex instability –
Katsanikas, Patsis, Contopoulos (2011, IJBC 21, 2321)



# Complex instability



# Complex instability – confined torus
# (structure-supporting only close to critical points)



- PERIODIC ORBITS:

  Do not exist in galaxies / snapshots of simulations
  Very important to know their morphology and stability.
  They structure the phase space

- NON-PERIODIC ORBITS:

  - Quasi-periodic: (regular)
    - useful (e.g., x1, x2)
    - not so useful (e.g., x4)
  - Chaotic:
    - Sticky chaotic
    - Strongly chaotic

# Summary: Two scenaria
# 1. The main

ORDER: Structures supported by quasi-periodic orbits

1. Bars. Outer 2D part and inner 3D part (peanut) – objections
2. Nuclear (x2), Inner, Outer rings in barred galaxies
3. Inner boxy isophotes in face-on images of bars
4. Spiral arms in non-barred galaxies

# 2. The alternative

CHAOS: Sticky Chaotic orbits (for times important in Galactic Dynamics)

1. Spirals (and rings) in barred-spiral systems
2. Envelopes of bars
3. Contribution to peanuts and inner boxy isophotes of face-on images of barred galaxies

# Details…

- Normal (non-barred) spirals are associated with "precessing ellipses" flows
- Tightly wound normal spirals (Sa) are weak (1-2% of the axisymmetric background in forces) and rotate fast. They may cross corotation.
- Open normal spirals (Sb-Sd) are strong (5-12% of the axisymmetric background in forces) and rotate slowly. Their symmetric part does not reach corotation.
- Spirals in barred-spiral systems may be "chaotic spirals" (Sba) or they may follow a "precessing ellipses" flow (SBb, SBc)
- In "precessing ellipses" flows, inside corotation, star formation is taking place ahead of the spiral arms, relative to the direction of rotation.
- Star formation in "chaotic spirals" flows is an open issue.
- Star forming regions are associated with rings (nuclear, inner, outer), i.e. with the location of resonances.
- Rings act as barriers that impede the inward flow of the gas toward the centers of galaxies.
- Close to the galactic center the Dynamics are dominated by gas.

Thank you !